

# Applications of Geometric .....Sef]Ygto Financy

Content Course 4.3 & 4.4



Development Team

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## Question 1

AER before DIRT

Using one of the brochures for NTMA State Savings products, as shown below, verify the Gross AER values, given the Total Return. (See columns headed "Actual before DIRT").

NTMA Brochure 1  
16 December 2012

The main NTMA State Savings™ products are:

Actual BEFORE DIRT		NTMA State Savings™ Products	Actual AFTER DIRT	
Total Return	Gross AER <sup>1</sup>		Net AER <sup>1</sup>	Total Return
	0.25%	Ordinary Deposit Account (DEMAND) <sup>5</sup>	0.17%	
	1.00%	Deposit Account Plus (90 DAY NOTICE) <sup>5</sup>	0.67%	
		Prize Bonds (WEEKLY DRAW) <sup>6</sup>		
7%	2.28%	3 year Savings Bonds <sup>2</sup>	2.28%	7.00%
12%	2.87%	4 year National Solidarity Bonds <sup>3</sup>	2.57%	10.68%
15%	2.83%	5 year Savings Certificates <sup>2</sup>	2.83%	15.00%
17%	2.90%	6 year Instalment Savings <sup>2,4</sup> (ALSO FOR CHILD BENEFIT)	2.90%	17.00%
45%	3.79%	10 Year National Solidarity Bonds <sup>3</sup>	3.55%	41.70%

<sup>1</sup> AER = "Annual Equivalent Rate"  
<sup>2</sup> Tax Free – Not subject to tax in Ireland.  
<sup>3</sup> Partial Tax – Annual interest of 1% is subject to DIRT (33% Jan 2013) – bonus is tax free. AER calculated on the assumption that the recipient of the annual NSB interest does not reinvest it.  
<sup>4</sup> Save in 12 monthly instalments (Max. €1,000 per month) and leave for 5 years. AER calculation assumes a 5% year average life.  
<sup>5</sup> Subject to DIRT = Deposit Interest Retention Tax (33% from January 2013).  
<sup>6</sup> Prize Bonds - Prize Fund is 2.25% pa. One €1 MILLION prize awarded each month, 1,000's of other weekly cash prizes (tax free in Ireland).

DIRT : 33% from January 2013

## Question 2

AER after DIRT

### 3. National Solidarity Bond 4 year (2nd Issue)

#### 4 Year National Solidarity Bond

Gross before DIRT	New 4 Year National Solidarity Bond	Net after DIRT
4.00%	Annual 1% interest payments over 4 years	2.68%
8.00%	The tax free bonus at the end of 4 years	8.00%
<b>12.00%</b>	<b>Total return</b>	<b>10.68%</b>
2.87%	The AER (Annual Equivalent Rate) is	2.57%

Minimum Savings - €100

Maximum Savings - €250,000 per individual

### 4. National Solidarity Bond 10 year (2nd Issue)

#### 10 Year National Solidarity Bond

Gross before DIRT	10 Year National Solidarity Bond	Net after DIRT
10.00%	Annual 1% interest payments over 10 years	6.70%
35.00%	The tax free bonus at the end of 10 years	35.00%
<b>45.00%</b>	<b>Total return</b>	<b>41.70%</b>
3.79%	The AER (Annual Equivalent Rate) is	3.55%

- The gross 1% annual interest is subject to the prevailing DIRT rate (33% in Jan 2013) - NET 0.67% p.a.

- AER calculated on assumption that the saver does not reinvest their annual 1% interest payment.

Minimum Savings - €100

Maximum Savings - €250,000 per individual

Given the information in the table opposite, and using the fact that the DIRT rate is 33% from January 2013, explain the origin of the following figures from the table:

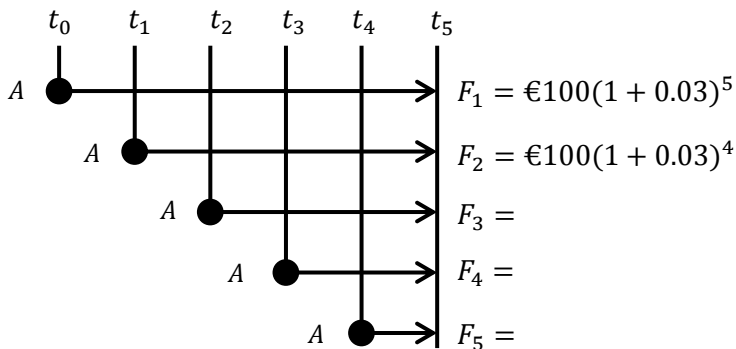
- (a) 2.68%
- (b) 10.68%
- (c) 2.75%
- (d) 6.7%
- (e) 41.7%
- (f) 3.55%

**Question 3 Future Value**

**(Annuity due)**

Five payments, each of €100, are paid into an account at regular yearly intervals, at the beginning of each of five years, at an AER of 3%, paid and compounded annually.

- (a) On the diagram below, fill in the future values for each of the regular payments of €100. The future values of the first two payments have been filled in for you.



- (b) Fill in the following table for the future value of the annuity at different times  $t_n$ .

$t_n$	Future value at $t_n$
$t_0$	€100
$t_1$	€100 + €100(1 + 0.03)
$t_2$	
$t_3$	
$t_4$	
$t_5$	

- (c) Calculate the future value of the annuity at the end of 5 years, using two methods.
- (d) Generalise the above procedure, to find the future value of five payments of € $A$ , paid into an account at regular yearly intervals, at the beginning of each of five years, where  $i$  is the AER expressed as a decimal, and the interest is paid and compounded annually.

**Question 4**

**Future value (ordinary annuity and comparison with annuity due)**

- (a) What is the future value of five payments, each of € $A$ , paid into an account at regular yearly intervals, at the end of each of five years, where  $i$  is the AER expressed as a decimal?  
(Draw and fill in, a timeline and a table for the situation, as in Q3. above.)
- (b) Compare the future values after five years, where the payments are made at the beginning of each year as in Q3., to the future values after five years, where the payments are made at the end of each year, as in this question.
- (c) If payments were made at the beginning of each of the five years and the last payment was made on the day that the investment matured, how many payments would have been made into the account?

**Question 5****Different compounding periods**

Calculate the number of compounding periods in the given time intervals, as the length of the compounding period varies.

Time intervals/years	Number of Compounding Periods			
	Interest added every 6 months	Interest added every 3 months	Interest added every month	Interest added every day*
3.5				
10				
40				

\* Ignore leap years.

**Question 6****Regular savings (Future value of an annuity)**

At the end of each month a deposit of €500 is made into an account that pays an AER of 8% paid and compounded monthly. Calculate the final amount (future value) after 5 years?

**Question 7****Regular savings (Future value of an annuity plus lump sum compounding)**

Sonya deposits €300 at the end of each quarter into her savings account.

- (a) If the money earns 5.75% (EAR), how much will this investment be worth at the end of four years?
- (b) After four years she is offered 6% AER, paid and compounded annually, if she does not withdraw the money for the following two years. How much will her investment amount to at the end of the six years?

**Question 8****Regular savings (Future value of an annuity)**

How long should an investor continue to make monthly investments of €1000 at a rate of 3% AER, paid and compounded annually, if she wishes to have at least €100 000 in a savings account? Assume that the first payment is made immediately and that the last payment is made on the day that the investment matures.

**Question 9****Saving for College fees**

Ben wants to save some money so that he can have €40 000 saved for his unborn daughter on her 18<sup>th</sup> birthday to help with college fees. On the day that his daughter is born, he begins making equal monthly payments into an account that pays 3.5% AER which is paid and compounded monthly.

His last payment into the account is due one month before his daughter turns 18.

Calculate the monthly payment required to achieve the fund of €40 000 in the time given.

**Question 10****More than one annuity**

Ailish deposits €200 at the beginning of each month for four years, into a savings account which pays 2.5% AER paid and compounded monthly. At the end of each year she also deposits €1000 into this account. How much money will she have in the account at the end of the 4 years?

**Question 11****Sinking fund**

Companies often purchase equipment and use it for a specified time period. The old equipment is then sold at scrap value and new, upgraded equipment is bought. In order to finance the purchasing of the new equipment, the company sets up, in advance, an annuity called a sinking fund.

A university buys a bus that costs €105,000 and its useful life is 5 years. It depreciates at 12% p.a. reducing balance. Assume that the cost of this type of bus increases at 3% AER, compounded annually.

The old bus will be sold at scrap value in 5 years and the proceeds will be used together with a sinking fund to buy a new bus. The university will make payments each month into a savings account giving a 4.4% AER compounded monthly. The first payment will be made at the end of the first month and the last payment will be made at the end of the 5 year period.

- (a) What is the scrap value of the bus after 5 years?
- (b) Calculate the cost of the a new bus of the same type, in 5 years' time.
- (c) Calculate the amount required in the sinking fund.
- (d) Calculate the monthly repayments required for the purchase of the new bus.

**Question 12****Sample Paper 1 LCHL Project Schools 2011**

Pádraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

- (a) Write down the present value of a future payment of €20,000 in one years' time.
- (b) Write down, in terms of  $t$ , the present value of a future payment of €20,000 in  $t$  years' time.
- (c) Pádraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value on the date of retirement of the fund required.
- (d) Pádraig plans to invest a fixed amount of money every month in order to generate the fund calculated in part (c). His retirement is  $40 \times 12 = 480$  months away.
  - (i) Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 3%.
  - (ii) Write down, in terms of  $n$  and  $P$ , the value on the retirement date of a payment of € $P$  made  $n$  months before the retirement date.
  - (iii) If Pádraig makes 480 equal monthly payments of € $P$  from now until his retirement, what value of  $P$  will give the fund he requires?
- (e) If Pádraig waits for ten years before starting his pension investments, how much will he then have to pay each month in order to generate the same pension fund?

Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an *annual-payments* option or a *cash-value* option. In the case of the *New York Lotto*, there are 26 annual payments in the *annual-payments* option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The *cash-value* option pays a smaller amount than this.

- (a) If the amount of the first annual payment is  $A$ , write down, in terms of  $A$ , the amount of the second, third, fourth and 26th payments.

1st payment (now):      $A$     

2nd payment:                     

3rd payment:                     

4th payment:                     

                  :   :

26th payment:                     

- (b) The 26 payments form a geometric series. Use this fact to express the advertised jackpot prize in terms of  $A$ .
- (c) Find, correct to the nearest dollar, the value of  $A$  that corresponds to an advertised jackpot prize of \$21.5 million.
- (d) A winner who chooses the *cash-value* option receives, immediately, the total of the present values of the 26 annual payments. The interest rate used for the present-value calculations is 4.78%. We want to find the cash value of the prize referred to in part (c).

- (i) Complete the table below to show the actual amount and the present value of each of the first three annual payments.

payment number	time to payment (years)	actual amount	present value
1	0		
2	1		
3	2		

- (ii) Write down, in terms of  $n$ , an expression for the present value of the  $n$ th annual payment.
- (iii) Find the amount of prize money payable under the *cash-value* option. That is, find the total of the present values of the 26 annual payments.  
Give your answer in millions, correct to one decimal place.
- (e) The jackpot described in parts (c) and (d) above was won by an Irish woman earlier this year. She chose the *cash-value* option. After tax, she received \$7.9 million.  
What percentage of tax was charged on her winnings?

**Question 14****Amortised Loan**

Sean borrows €10 000 at an APR of 6%.

He repays it in five equal instalments of €2373.96 over five years, with the first repayment one year after he takes out the loan.

Fill in the amortisation schedule below.

Payment #	Fixed payment	Interest portion	Principal portion	Balance
0				€10,000.00
1	€ 2373.96	€600.00	€1773.96	€ 8226.04
2	€ 2373.96			
3	€ 2373.96			
4	€ 2373.96			
5	€ 2373.96			€0
<b>Totals</b>				

<b>Rate per year</b>	6.00%
<b>years</b>	5
<b>fixed payment</b>	€2373.96

- (a) How is the interest portion of each successive payment changing? Why?
- (b) How is the principal portion of each successive payment changing? Why?
- (c) What do the principal portions of all the payments add up to?
- (d) What do the present values of all the payments add up to?

**Question 15****(a) Amortised Loan (b) Bonds**

- (a) A graduate is setting up his own company. He borrows the €5000 for set-up costs for 6 months at a flat rate of 1% per month (compounded monthly). He wants to arrange to pay this off in equal monthly instalments.
- (i) Calculate the monthly repayment amount.
- (ii) Make a schedule showing the monthly payment, the monthly interest on the outstanding balance, the portion of the payment contributing toward reducing the debt, and the outstanding balance.
- (b) After 5 years the company needs to raise money to expand. It proposes to issue a 10-year €2000 bond that will pay €100 every year. If the current market interest rate is 5% per annum, what is the fair market value of this bond? Explain your answer and justify any assumptions you make.

**Question 16****Applying skills learned in previous questions**

Paul applies for a loan of € 95,000. On the website he enters his details into the loan calculator:

Amount you would like to borrow (in Euro)?

OR

What monthly repayment can you afford?

Interest rate (APR)?  %

Over what term do you want to repay the loan?

The calculator produced the following result, giving his monthly repayments and his total cost of credit (interest payable)

Loan Calculator Results	
Monthly repayment:	<input type="text" value="A"/>
Loan amount:	€95,000
Interest rate:	3.4%
Term:	15 years
Cost of Credit:	<input type="text" value="B"/>

- Calculate the monthly interest rate.
- Calculate the monthly repayment, labelled A on the loan calculator.
- Make a schedule, showing the monthly payment, the monthly interest on the outstanding balance, the portion of the payment contributing towards reducing the debt and the outstanding balance (for the first 6 months).
- Show how to use the sum of a geometric series to calculate the amount Paul has paid off his loan after 4 months. (Assume that he makes his first payment at the end of the first month.)
- Calculate the total interest Paul will pay on the loan, labelled B on the loan calculator.
- After 5 years Paul decides to reduce the term of his loan. He can afford to put an extra €100 per month towards his loan repayments. How many years will it take him to pay off his loan?
- How much will Paul have saved on his mortgage by making this change?
- A bank is offering a regular monthly savings account with an AER of 4% on balances up to €15 000. The interest earned is subject to tax at a rate of 30%. Paul needs to decide whether it would be a better option to invest €100 per month into this savings account or to pay the €100 extra into his loan account after 5 years.
  - If Paul were to save €100 per month, starting today, show how to use the sum of a geometric series to calculate the future value of his investment over the length of time of his re-adjusted mortgage term.
  - Calculate the amount of interest Paul will earn on his investment over the lifetime of his re-adjusted mortgage.
  - Which is Paul's best option?



**Question 17****Present value**

Suppose that you receive annual payments of €10 000 per annum for a period of 10 years. What is the present value of this stream of payments, if the first payment will be made in 4 years time and the interest rate is 5%.

**Question 18****The Lotto**

The €40 million lottery you just won actually pays €2 million per annum for 20 years. If the discount rate is 8% and the first payment is made in 1 year, what is the present value of the winnings? What if the first payment is paid immediately?

Sean borrows €10,000 at an APR of 6%.

He repays it in five equal instalments of €2373.96 over five years, with the first repayment one year after he takes out the loan.

Present values of repayments	$P_5$	$P_4$	$P_3$	$P_2$	$P_1$	$\sum_{n=1}^y P_n$	Amount paid off the capital sum each year.
Outstanding balance at time 0.	$\frac{2373.96}{(1.06)^5}$ = €1773.96	$\frac{2373.96}{(1.06)^4}$	$\frac{2373.96}{(1.06)^3}$	$\frac{2373.96}{(1.06)^2}$	$\frac{2373.96}{(1.06)^1}$	= €10000	
Pay €2373.96	Outstanding balance at end of year 1.	$\frac{2373.96}{(1.06)^4}$ = €1880.40	$\frac{2373.96}{(1.06)^3}$	$\frac{2373.96}{(1.06)^2}$	$\frac{2373.96}{(1.06)^1}$	= €8226.04	-€1773.96
	Pay €2373.96	Outstanding balance at end of year 2.	$\frac{2373.96}{(1.06)^3}$ = €1993.23	$\frac{2373.96}{(1.06)^2}$	$\frac{2373.96}{(1.06)^1}$	= €6345.63	-€1880.40
Pay €2373.96	Pay €2373.96	Outstanding balance at end of year 3.	$\frac{2373.96}{(1.06)^2}$ = €2112.82	$\frac{2373.96}{(1.06)^1}$	$\frac{2373.96}{(1.06)^1}$	= €4352.41	-€1993.23
	Pay €2373.96	Pay €2373.96	Outstanding balance at end of year 4.	$\frac{2373.96}{(1.06)^1}$	$\frac{2373.96}{(1.06)^1}$	= €2239.58	€2112.82
		Pay €2373.96	Outstanding balance at end of year 5.	Pay €2373.96	Outstanding balance at end of year 5.	= €0	-€2239.58