Topic: Introduction to Theorem 19

Year Group: 3rd Year HL

For the lesson on 25th February2016
AT Coolmine Community School
Teacher: Micheál Gaffey
Lesson plan developed by: Deirdre Kelly, Joanna Garry, Orla Skelley, Micheál Gaffey

Title of the Lesson:

The introduction of Theorem 19

Brief description of the lesson:

To help the students realize that the angle at the centre of a circle standing on a given arc is twice the angle at any point on the circle standing on the same arc. Students should also discover the conditions in which this is not the case.

Aims of the Lesson:

That students understand the relationship between the angle at the centre of a circle and the angle on the circle standing on the same arc.

Long-range/thematic goals:

I'd like my students to appreciate that by drawing and measuring geometric shapes that they can discover mathematical rules.
I'd like my students to recognise that there are different ways to tackle a given problem.
I'd like my students to appreciate that mistakes are part of the learning process and that they are not a negative thing.
I'd like to foster my students to become independent learners.
I'd like my students to become more creative in developing connections in Maths.
I'd like my students to find their maths lessons engaging, stimulating and challenging.
I'd like my students to understand that discussing ideas is an important part of Mathematics.
I'd like to build my students' enthusiasm for the subject by engaging them with stimulating activities
I'd like my students to connect and review the concepts that we have studied already

Lesson specific aims

I'd like my students to be able to measure an angle using a protractor.
I'd like my students to become fluent in the terminology and keywords used with regards to a circle.
I'd like my students to be able to learn from each other by working in pairs.

I'd like my students to discover that the angle at the center of a circle standing on a given arc is twice the angle at any point on the circle standing on the same arc.

Learning Outcomes:

As a result of studying this topic students will be able to:
Understand the meaning of:

- An angle at the centre of the circle
- An angle on the circle

Understand the relationship between these two angles
Understand fully the conditions under which this relationship exists.
Understand fully the conditions under which this relationship does not exist.

Background and Rationale

This theorem has many corollaries so we really feel that this approach is fundamental to their understanding of the geometry of a circle. By giving the students an opportunity to discover, we are allowing students to understand the geometry of the circle. Theorem 19. The angle at the centre of a circle standing on a given arc is twice the angle at any point of the circle standing on the same arc. Proof. There are several cases for the diagram. It will be sufficient for students to examine one of these. The idea, in all cases, is to draw the line through the centre and the point on the circumference, and use the Isosceles Triangle Theorem, and then the Protractor Axiom (to add or subtract angles, as the case may be).

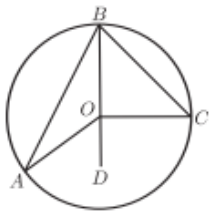


Figure 24.

In detail, for the given figure, Figure 24, we wish to show that $|\angle AOC| = 2|\angle ABC|$.

Join B to O and continue the line to D. Then

$$\begin{aligned} |OA| &= |OB|. && \text{[Definition of circle]} \\ \therefore |\angle BAO| &= |\angle ABO|. && \text{[Isosceles triangle]} \\ \therefore |\angle AOD| &= |\angle BAO| + |\angle ABO| && \text{[Exterior Angle]} \\ &= 2 \cdot |\angle ABO|. \end{aligned}$$

Similarly,

$$|\angle COD| = 2 \cdot |\angle CBO|.$$

Thus

$$\begin{aligned} |\angle AOC| &= |\angle AOD| + |\angle COD| \\ &= 2 \cdot |\angle ABO| + 2 \cdot |\angle CBO| \\ &= 2 \cdot |\angle ABC|. \end{aligned}$$

Corollary 2. *All angles at points of the circle, standing on the same arc, are equal. In symbols, if A, A', B and C lie on a circle, and both A and A' are on the same side of the line BC, then $\angle BAC = \angle BA'C$.*

Proof. Each is half the angle subtended at the centre. □

Remark 7. The converse is true, but one has to careful about sides of BC:

Converse to Corollary 2: *If points A and A' lie on the same side of the line BC, and if $|\angle BAC| = |\angle BA'C|$, then the four points A, A', B and C lie on a circle.*

Proof. Consider the circle s through A, B and C. If A' lies outside the circle, then take A'' to be the point where the segment $[A'B]$ meets s. We then have

$$|\angle BA'C| = |\angle BAC| = |\angle BA''C|,$$

by Corollary 2. This contradicts Theorem 6.

A similar contradiction arises if A' lies inside the circle. So it lies on the circle. □

Corollary 3. *Each angle in a semicircle is a right angle. In symbols, if BC is a diameter of a circle, and A is any other point of the circle, then $\angle BAC = 90°$.*

Proof. The angle at the centre is a straight angle, measuring 180°, and half of that is 90°. □

Corollary 4. *If the angle standing on a chord $[BC]$ at some point of the circle is a right angle, then $[BC]$ is a diameter.*

Proof. The angle at the centre is 180°, so is straight, and so the line BC passes through the centre. □

Research

We intend to build on student's knowledge with regards to triangles, simple isosceles and equilateral triangles, the angles within, and exterior angles. Furthermore, students should build on prior knowledge of the properties of the circle, knowing the terminology - centre; radius; arc (major and minor/small and big), etc. Finally, students will be invited to use their geometry sets independently to measure angles. Our lengthy discussions have led us to settle on the discovery approach to interpret concept and relationship between the angle at the centre of the circle and the angle on the circle standing on the same arc. The students will discover logical steps to aid in the proof of this theorem. We intend to use Geogebra to facilitate visual interpretation of the theorem and angles on the arc and to support teaching and learning in the lesson.

About the Unit and the Lesson

To start the lesson students will be presented with a simple task: identification of the circle and associated properties. Students will be invited to engage in a simple exercise in identification of the properties of a triangle which sits on the diameter. Students will then be invited to identify, through discover-led learning, the relationship between the angle at the centre of the circle and on the arc.

Flow of the Unit:

Lesson		# of lesson periods
1	**Introduction to Theorem 19**	1 x 40min

Flow of the Lesson

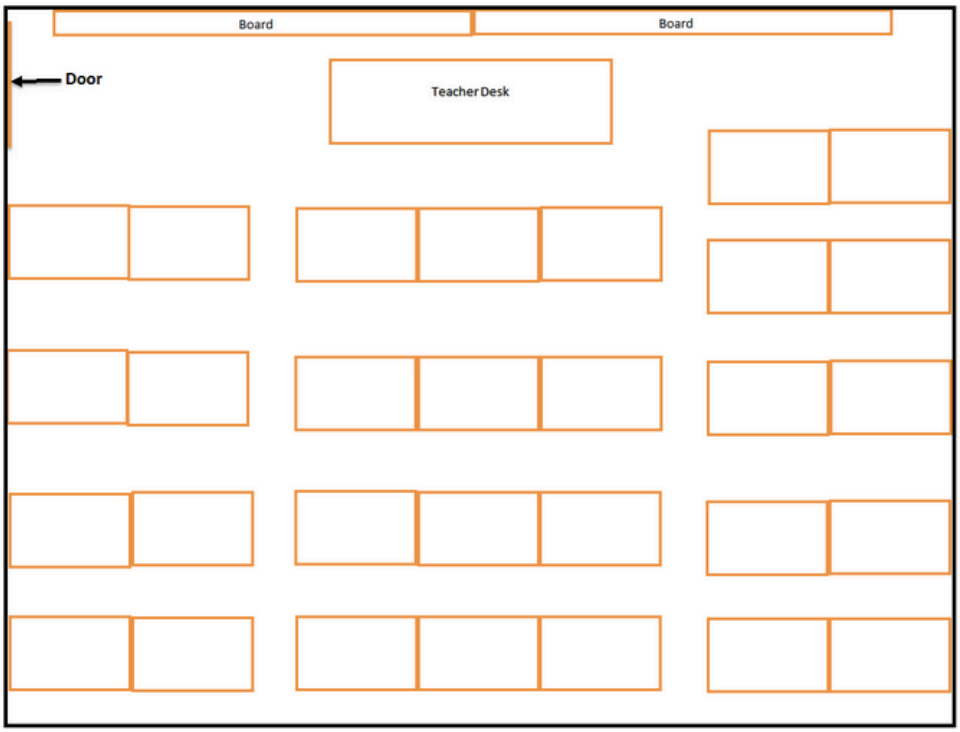
Teaching Activity	Points of Consideration
1. Introduction Teacher will use classroom whiteboard and Geogebra to recap on prior knowledge of circles, radii and angles.	Pupils may be able to explain the properties of circles and triangles displayed on the whiteboard.
2. Posing the Task 1. Invite students to draw 2 radii on a prepared worksheet with a series of blank circles 2. Direct students to identify those circles they have created where the two radii form an acute and obtuse angle. Students will measure and mark in the relevant measured in degrees. 3. Direct students to identify and mark the minor ("little/small") arc on the circle on which the two radii sit. 4. Students will now be directed to draw two lines to form an angle on the circle. They will be directed to start each line from the point at which the radius meets the arc and to direct lines away from the arc (examples given on Geogebra). 5. Students now use their protractors to measure all of the angles they have just created and are asked to consider any relationship they	How do we know if students understand the task? Can students draw a series of radii on a circle and understand a series of different possibilities exist? Can students identify the circles where acute and obtuse angles may be measured? Have they correctly measured the angles. Do students identify that there are two possible arcs? Can they correctly mark and label the minor arc? Do students recognize they can create an angle using the two points that the radii create where they meet the circle?

may have discovered. 6. Students are now directed to repeat this process, however they should not allow the angle formed to meet the circle in this second instance. Students are firstly directed to form an angle outside the circle and subsequently to form an angle inside the circle.	Can students determine that there is a relationship between the two angles when one occurs on the circle as opposed to inside/outside? Can students repeat process to create angles within/outside the circle.
3. Anticipated Student Responses Students should find the activity engaging as it is based on prior knowledge and involves them solving some basic problems. Most students should find the task fairly straightforward. Some students might find it difficult to correctly draw angles on the circle. Some students may find it difficult to draw alternative radii/identify major/minor arcs. Some students might find it difficult to explain their reasoning for drawing radii in specified locations. Some students will come up with their own language for describing how they created arc.	Use of Geogebra and whiteboard in combination with pre-prepared posters are of great benefit in this activity as they allow the teacher to monitor the progress of every student in the class. The teacher can target particular students for additional help or to share their thinking accordingly.
4. Comparing and Discussing Teacher identifies which student solutions should be shared with the class in order for students to compare their work. Teacher selects students whose work closely aligns with the anticipated responses developed in our pre-prepared posters. This activity follows the order of the lesson. Teacher posed open question at end of the lesson: "What have you noticed by doing these activities?"	Students are asked to reflect on their completed activities throughout the lesson and determine the relationship between the angle on the circle and that one the arc. Students can establish the same relationship does not exist when the angle is not on the circle. Student feedback and participation will indicate that students are benefiting from the discussion.
5. Summing up Ask the students to write a sentence or two on what they have learned today. Give the homework sheet and briefly explain the content of it.	It is anticipated that students will prepare sentences which demonstrate their newly acquired knowledge of theorem 19.

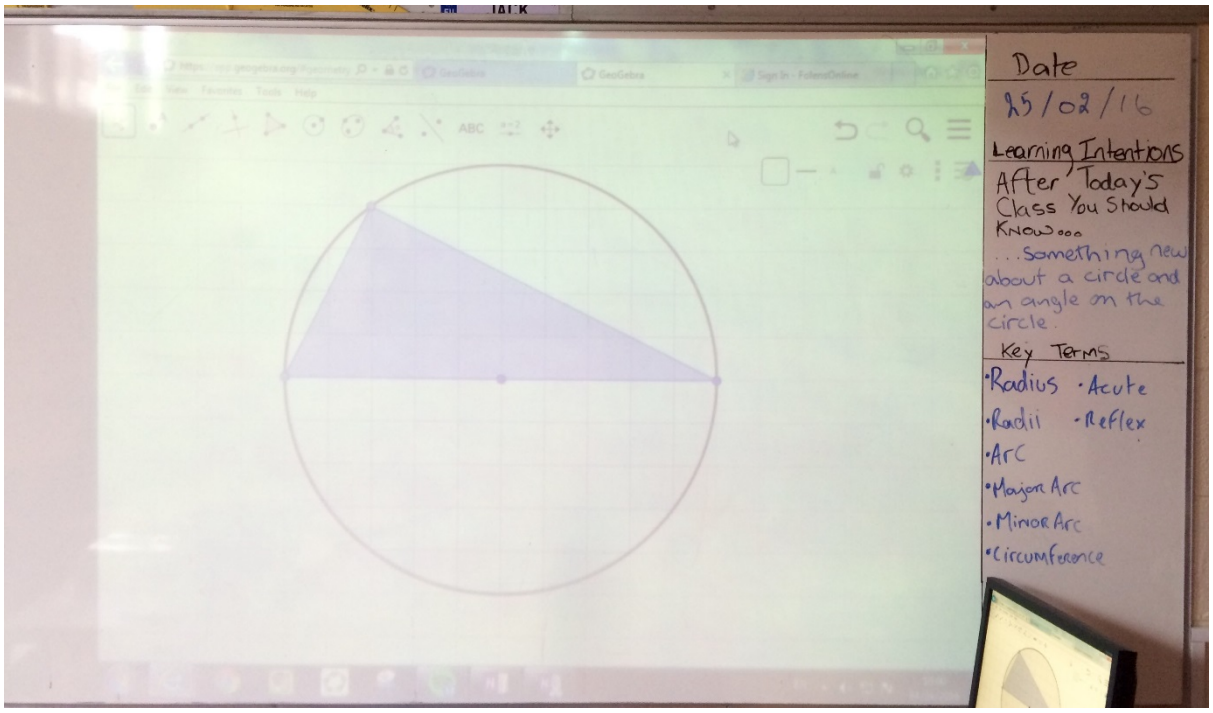
Evaluation

- Student work will be recorded in observation sheets and photographed throughout the class
- Three observing teachers will not interact with students, all instructions and direction will come directly from teacher, Micháel Gaffey. Three observing teachers will quietly photograph and record observations as class proceeds.
- Teacher will deal with any questions posed by students.
- Teachers will focus on engagement with task, peer to peer cooperation, pair work and feedback.
- Students' work will be gathered at close of lesson and statements evaluated for evidence of learning.

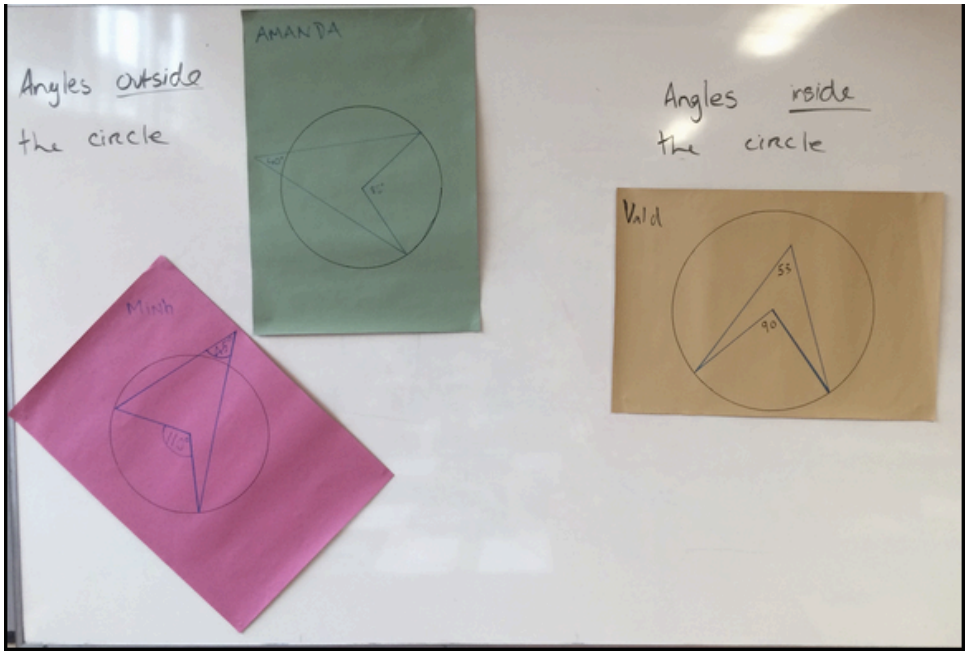
Classroom Layout

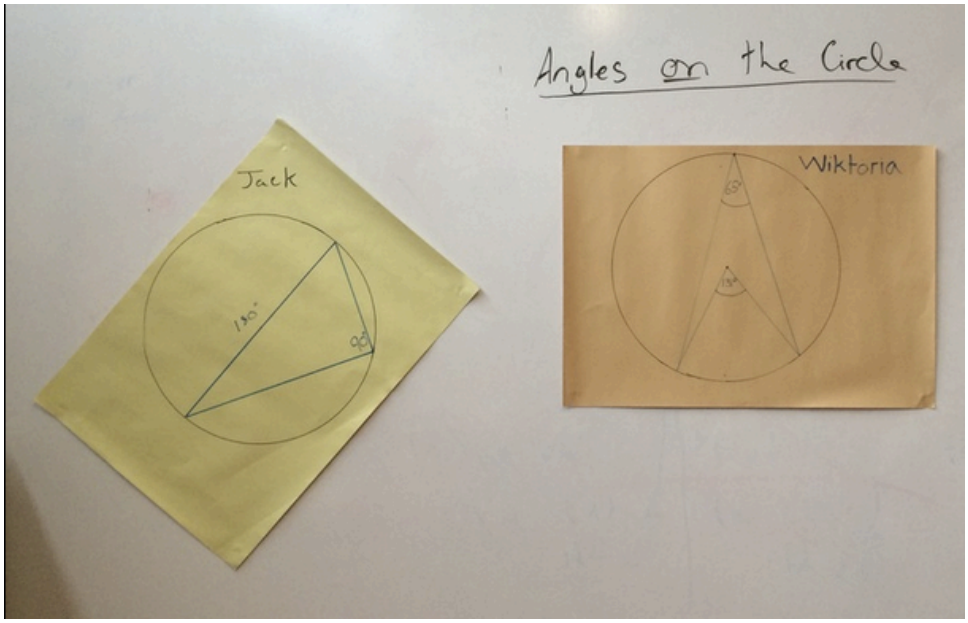


Board Plan



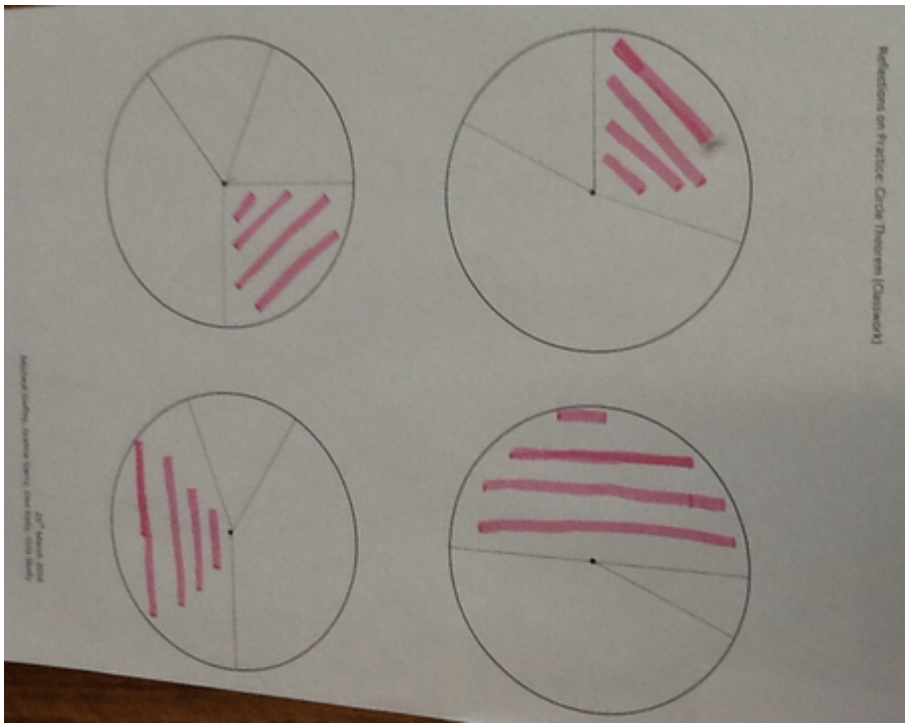
Board As The Lesson Progressed

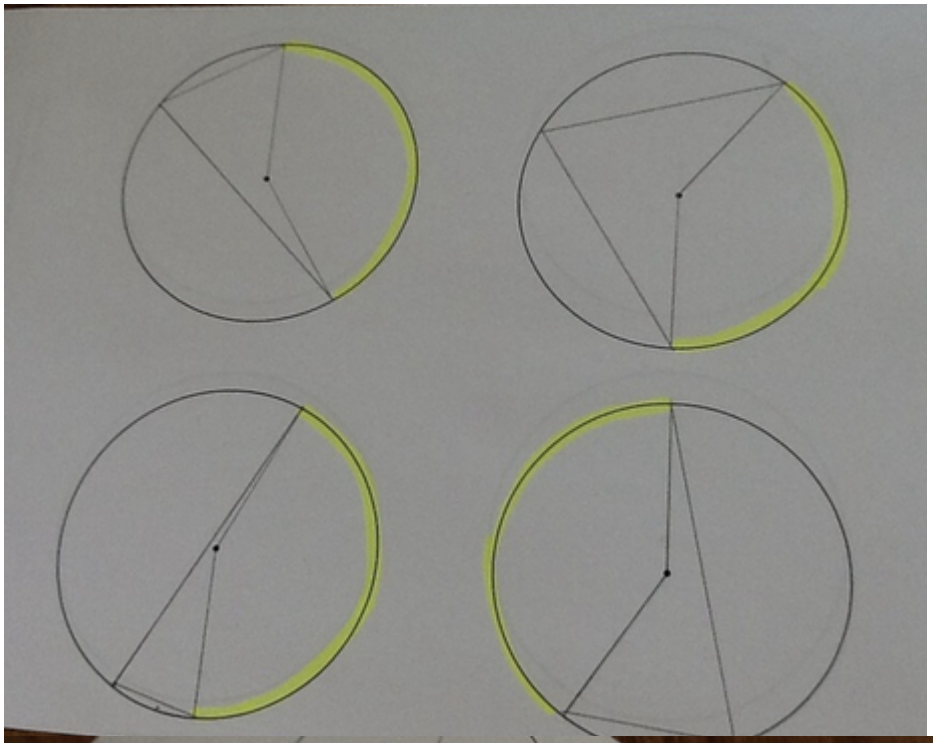


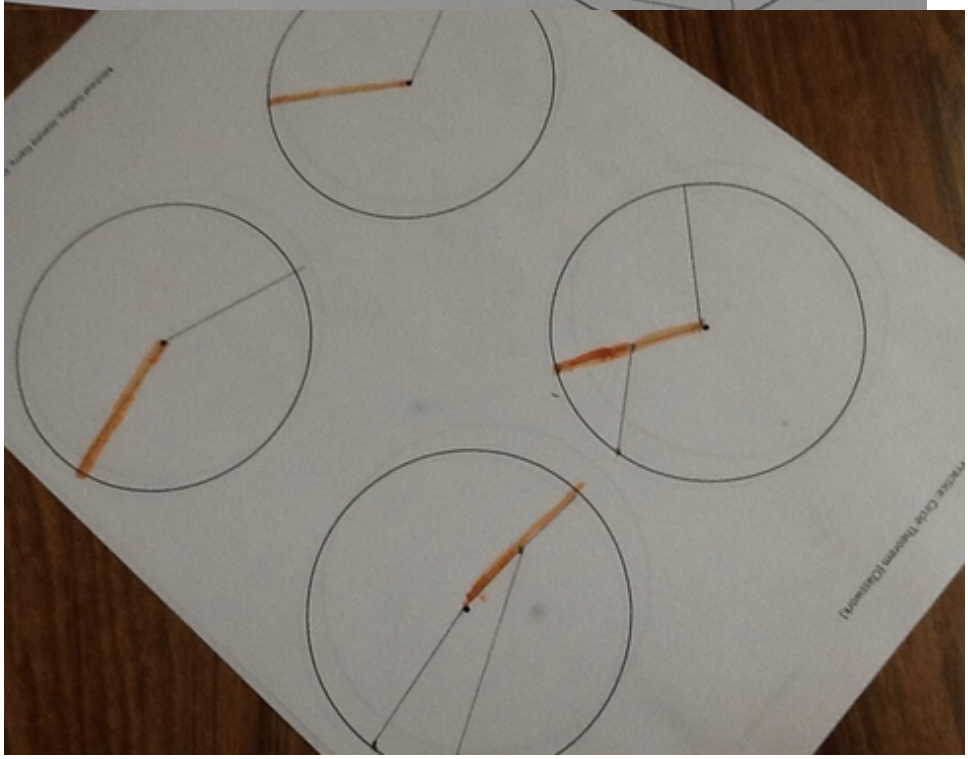


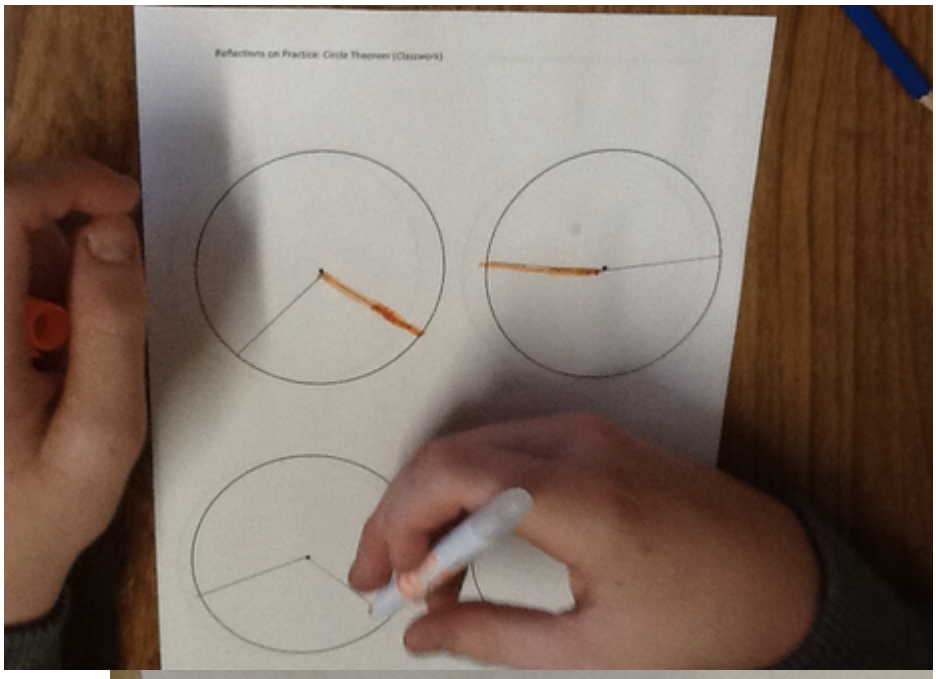
Examples of Students Work

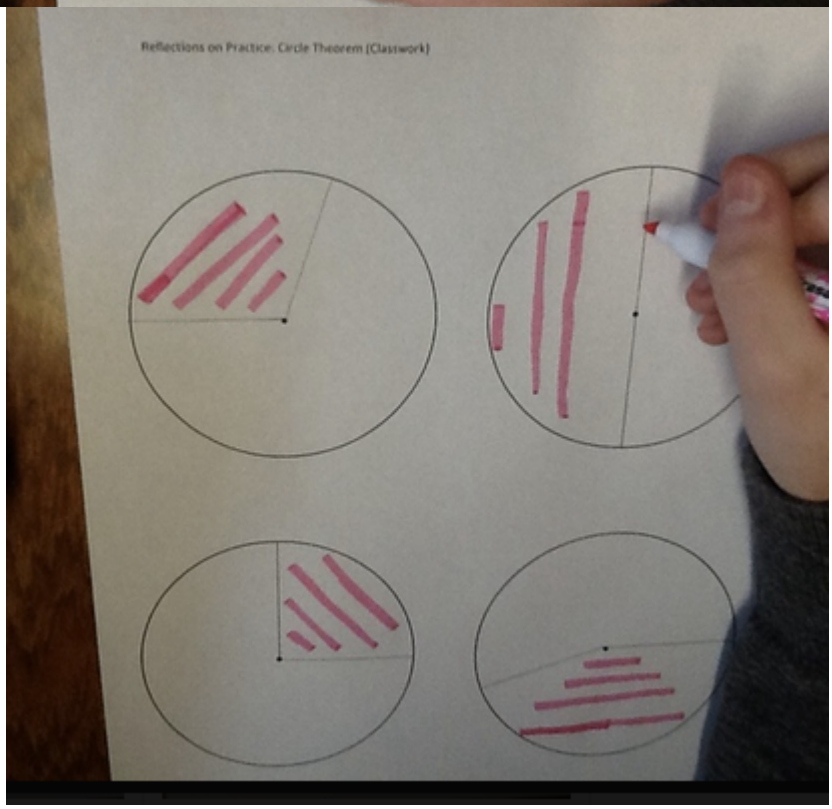
Some examples of where students misunderstood the verbal instructions








Reflections on Practice: Circle Theorem (Classwork)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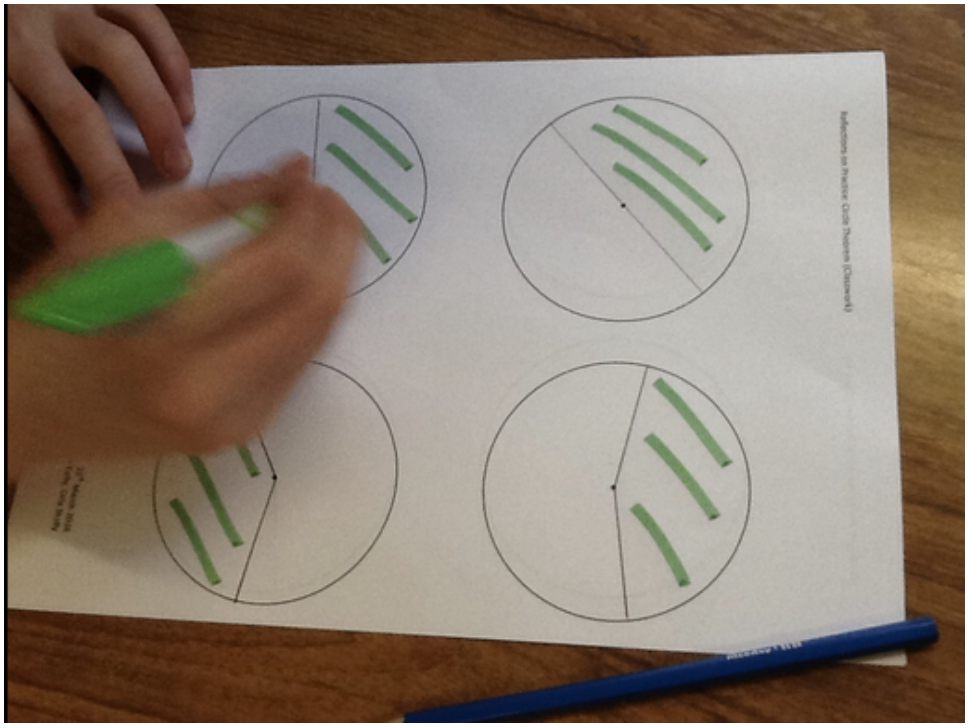
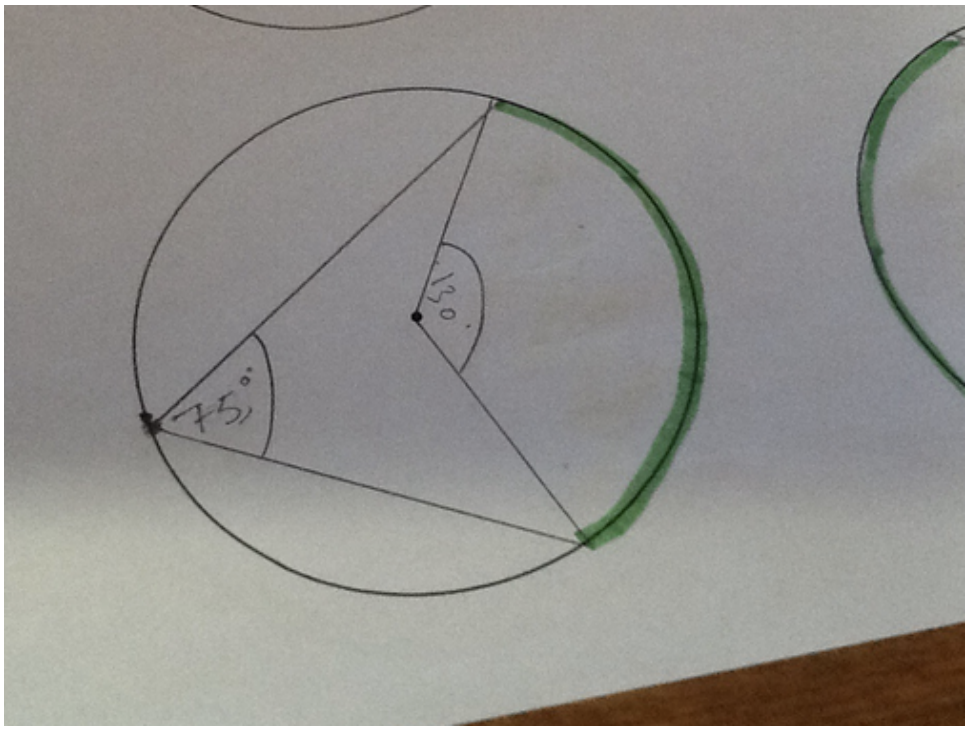
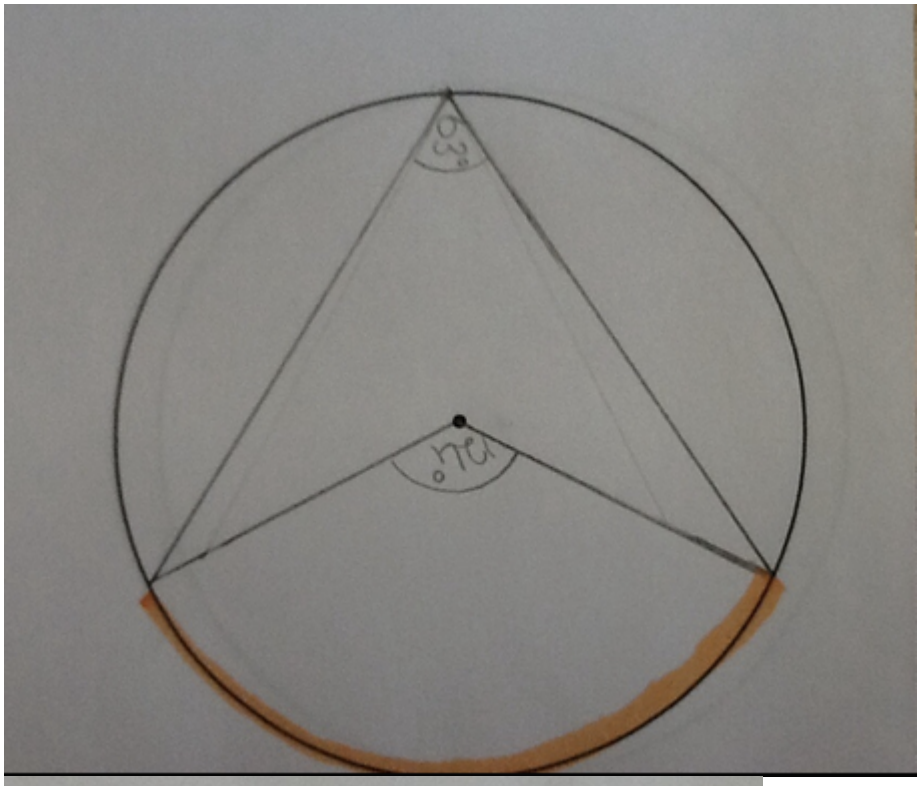
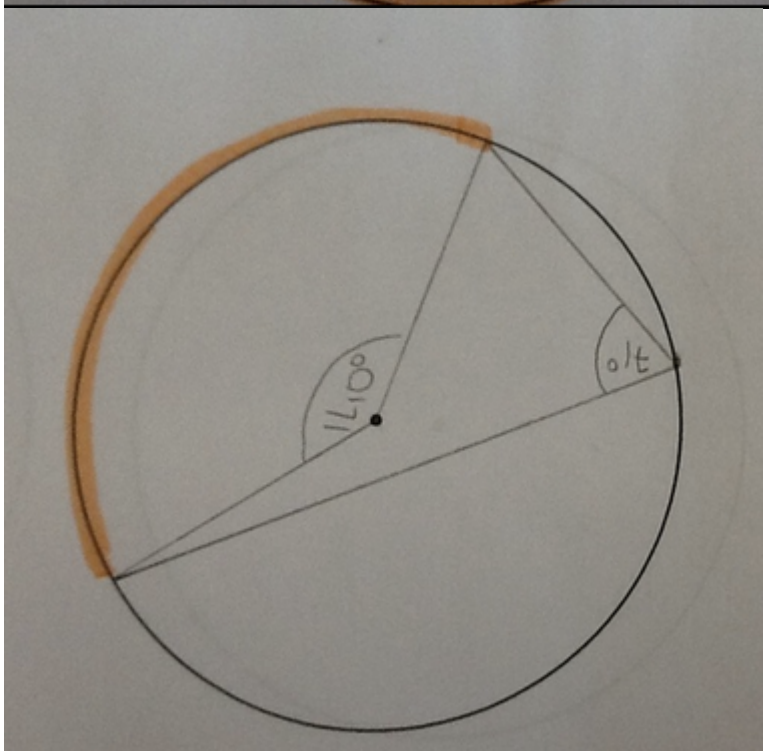
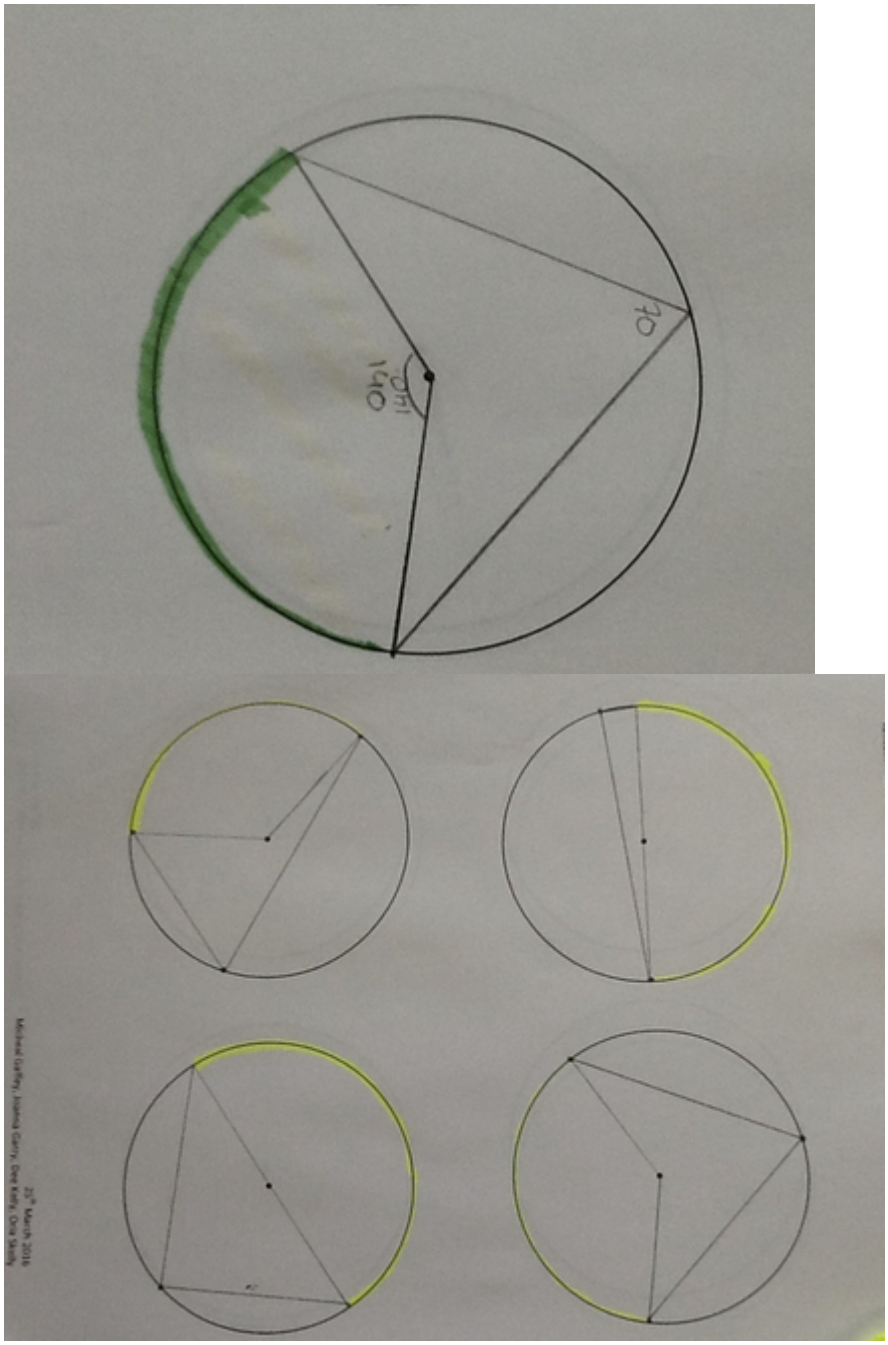


Task 1 (on the circle) - Some examples of students work

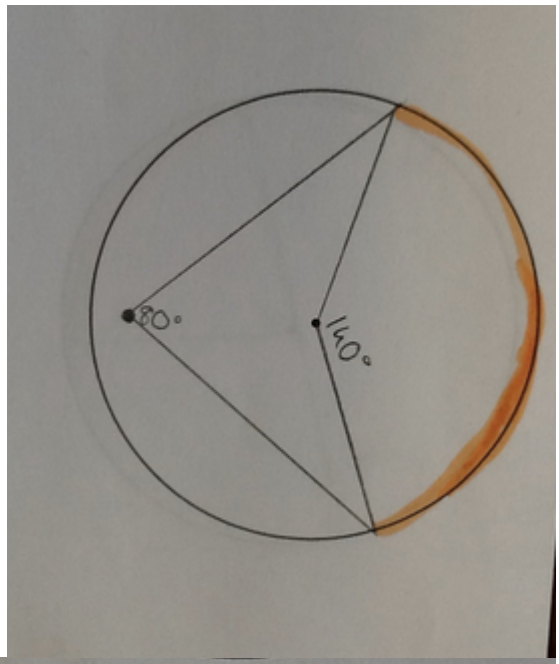


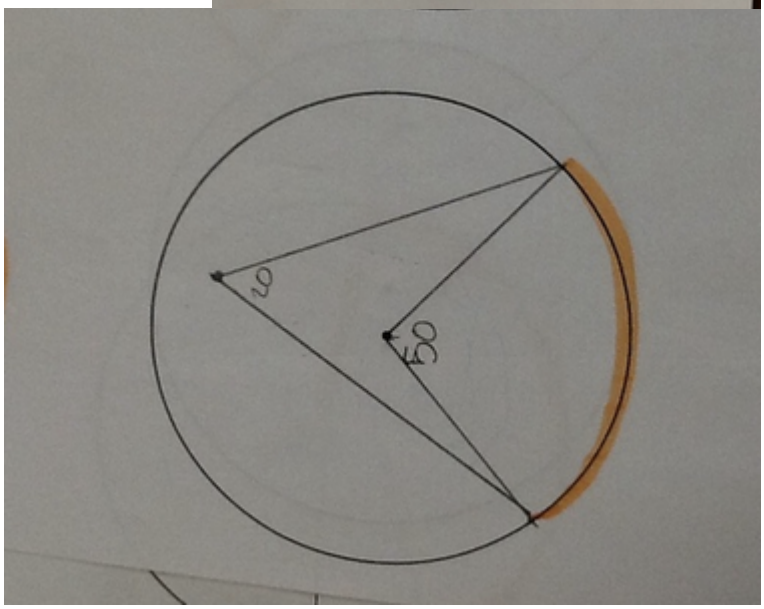

32°
124°

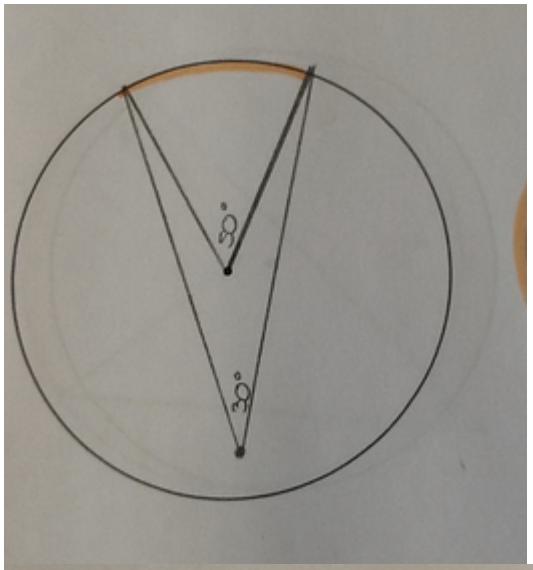

110°
41°

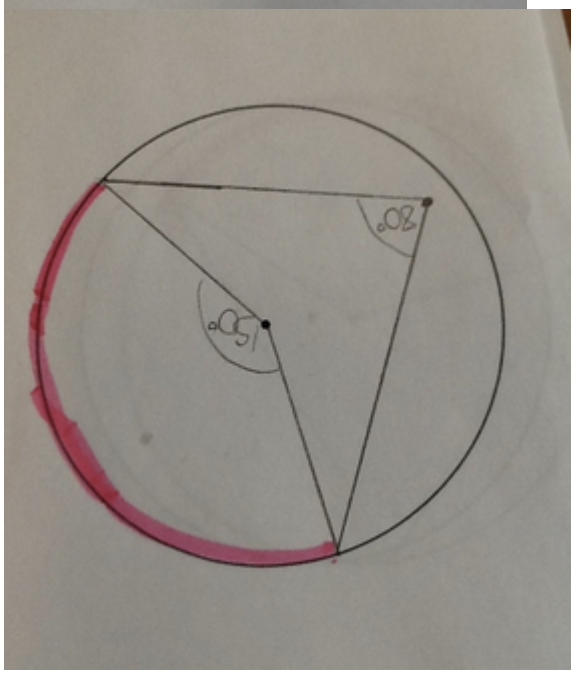


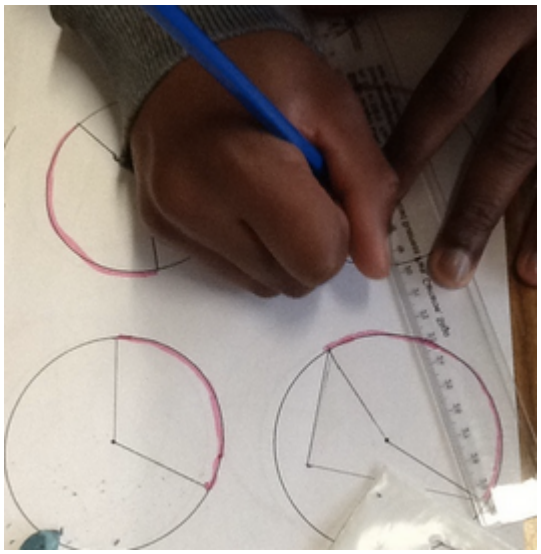
Task 2 (inside the circle) - Some examples of students work


80°
140°

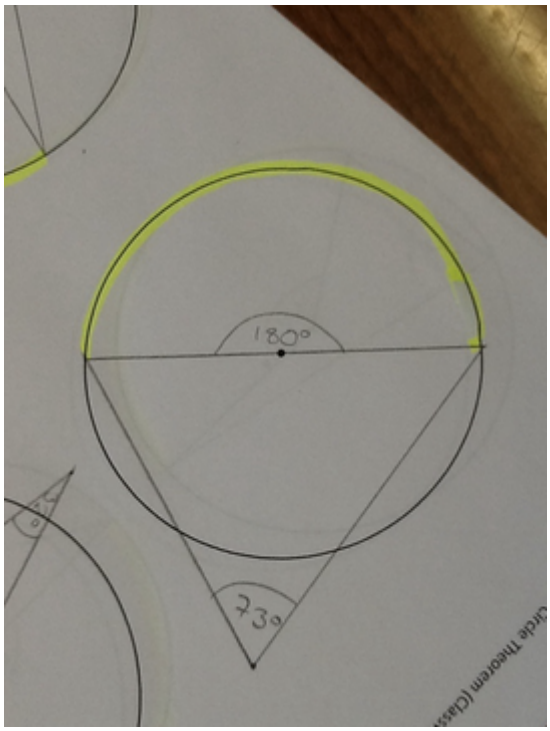

20
50

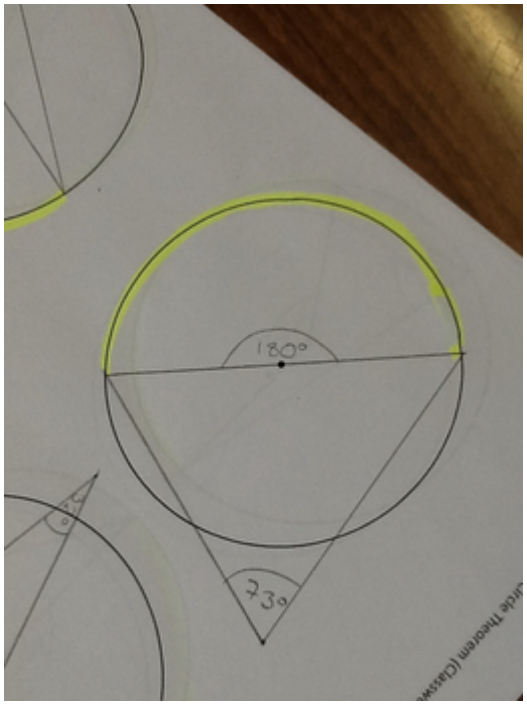

30°
30°

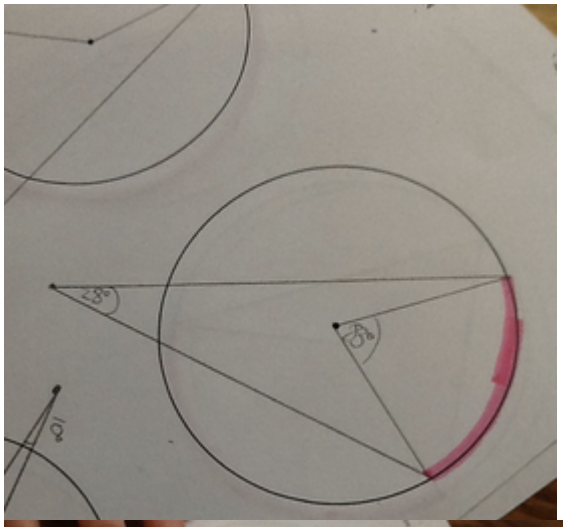

82°
105°



Task 3 (outside the circle) - Some examples of students work



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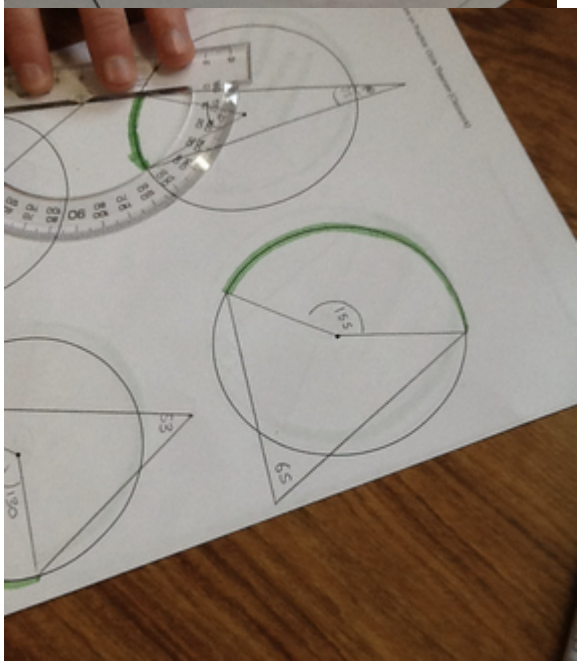
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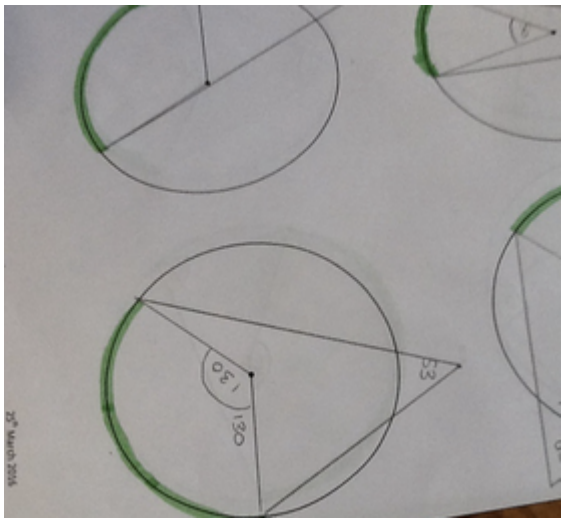
130


130
130
33
25th March 2014

Post-lesson reflection

- It was observed that students immediately engaged in task with enthusiasm, picking up pencils and rulers and drawing radii. Some students, working in pairs, drew just a single radius and required additional teacher instruction. This was a factor at lesson outset but students became very comfortable at a later stage; having completed the task once, they continued with ease and confidence. Having all equipment readily available on the students desks (pre-prepared worksheets, protractors, rulers, pencils etc) lent itself to successful and immediate engagement with task

- A number of students immediately copied example of triangle on diameter as presented on whiteboard. However, subsequent circles were composed of angles originating from radii in different locations. A significant improvement in confidence was observed. Evidence of peer to peer collaboration was strong and reaffirming. However, classroom layout could have been readjusted to allow the strong student support the weaker one.

- In highlighting minor arcs, many students instead shaded entire sectors or radius. Some intervention required by teacher. In addition many chords were drawn and lines drawn originating from the radius to meet the circumference. Again, correction by teacher was required.

- Verbal instruction was an issue for some students, the inquiry-led approach left some students unsure or lacking confidence in precisely how to proceed with the task. The teacher tackled this by using whiteboard and Geogebra visual aids to redirect students.

  Students were also invited, as the lesson proceeded, to consider if any relationship between the angles formed was evolving. Teacher encouraged students to take a step back and note their observations as they proceeded.

- As the lesson involved repetition of tasks to include minor modifications (angle formed within or outside circle), a huge increase in attainment of learning objectives was observed as lesson progressed.
  The activities designed to provoke learning greatly assisted students and it was observed that a recap of information gained in this lesson should be immediate in subsequent lesson.

- A small increase in available time would have been welcome.

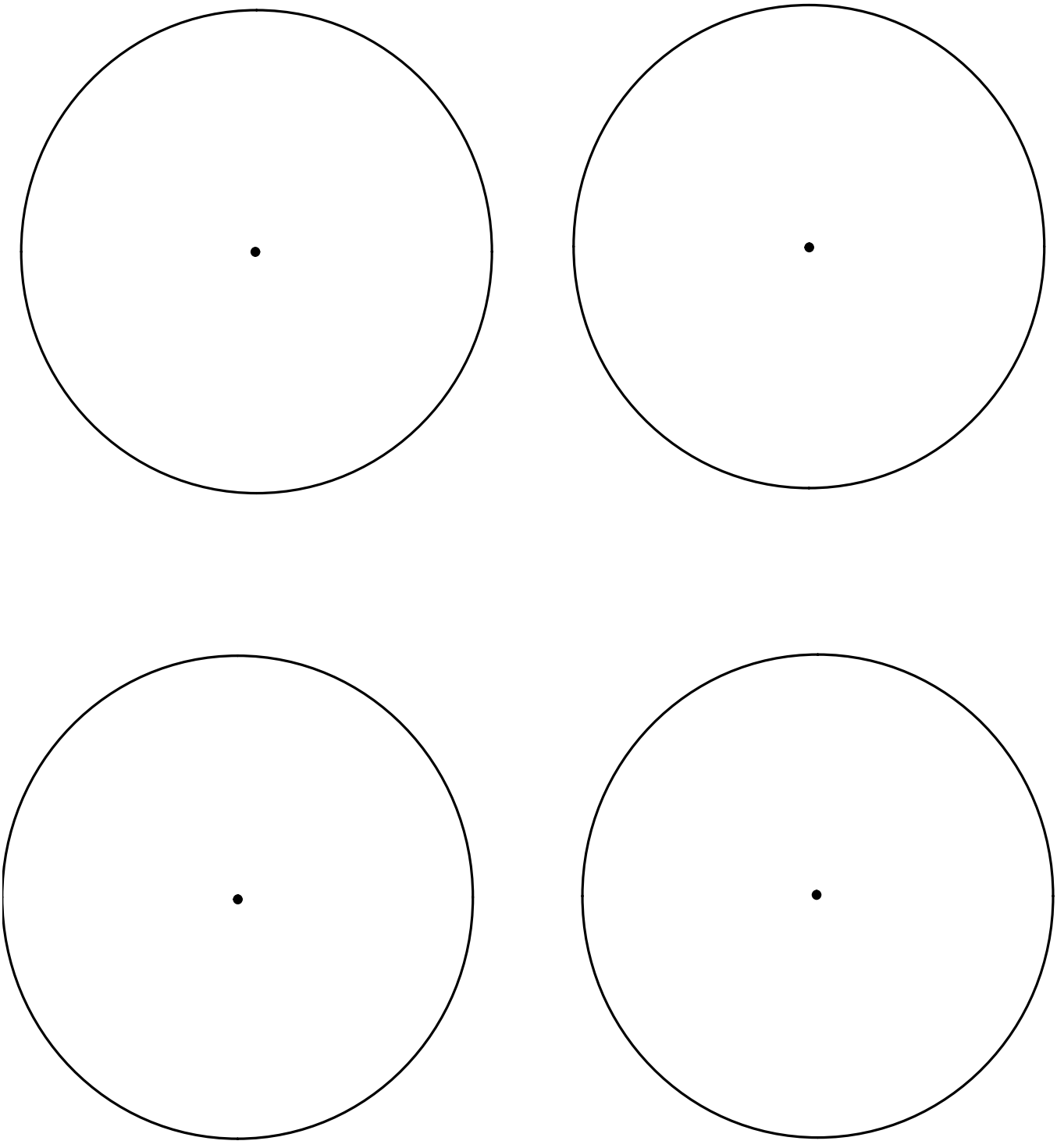
Appendices

A: Class worksheet

B: Homework sheet 1

C: Homework sheet 2

A



B

Now can you show when this relationship does not hold and explain why.

C

Do you notice any relationship between the different angles in the given diagram?

Now by using the other circles see if this is always the case.