

**Group:** Third Year Higher Level.

**Lesson Topic:** Geometry.

**Lesson Title:** Understanding and appreciating Axiom 4, and its value in geometrical problems.

For the lesson on 26th February 2016

St Louis Community School ,  
Third Year Higher Level class.

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**LESSON TITLE : UNDERSTANDING AND APPRECIATING AXIOM 4, AND ITS VALUE WHEN SOLVING GEOMETRICAL PROBLEMS.**

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This lesson is designed to allow students to appreciate Axiom 4 and the role it plays in geometry prior to the learning of the theorems so that they will be able to fully appreciate the axiom's role in formal proof. This is an area that we have identified as a crucial area to geometrical and logical thinking and one which has caused many students great difficulty.

**AIMS OF THE LESSON:**

The aims for students: (Thematic Goals)

- To appreciate that mathematics has contributed greatly through geometry to architecture and design
- To appreciate that mathematics can be used to communicate thinking effectively
- To become more creative when devising approaches and methods to solve problems
- To build enthusiasm for the subject by engaging them with stimulating activities and challenges
- To connect topics and develop strategies to investigate relationships and justify conclusions mathematically.

**LESSON SPECIFIC GOALS:**

Aims for students:

- Understand the term congruency at a deeper level than just equal triangles.
- Create congruent triangles and recognise their individual attributes
- To recognise that simple constructions of different types of line segments can create a unique set/pairs of congruent shapes
- The value and importance of being able to explain congruency and using congruency to justify their own geometrical decisions.

**LEARNING OUTCOMES:**

As a result of studying this topic students will be able to:

- Understand congruency can refer to any set of shapes
- Use axiom4 appropriately to provide reasoning to justify congruency(as per syllabus)
- Build upon axiom4 to provide reasoning when asked in a formal proof setting
- Identify when to use SSS, SAS, ASA, RHS and to be able to appreciate on certain occasions one or more of these conditions can be applied to prove congruency.

## BACKGROUND AND RATIONALE

We recognize that students are challenged by spatial reasoning and particularly by geometry problems in an unusual context. We have identified as a group that congruency is always an issue with students each year.

### Strand 2: Geometry and Trigonometry

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
<b>2.1 Synthetic geometry</b>	<p>Concepts (see <i>Geometry Course</i> section 9.1 for OL <b>and 10.1 for HL</b>)</p> <p>Axioms (see <i>Geometry Course</i> section 9.3 for OL <b>and 10.3 for HL</b>):</p> <ol style="list-style-type: none"> <li>[Two points axiom] There is exactly one line through any two given points.</li> <li>[Ruler axiom] The properties of the distance between points</li> <li>[Protractor Axiom] The properties of the degree measure of an angle</li> <li>Congruent triangles (SAS, ASA and SSS)</li> <li>[Axiom of Parallels] Given any line <math>l</math> and a point <math>P</math>, there is exactly one line through <math>P</math> that is parallel to <math>l</math>.</li> </ol>	<ul style="list-style-type: none"> <li>– recall the axioms and use them in the solution of problems</li> <li>– <b>use the terms: theorem, proof, axiom, corollary, converse and implies</b></li> </ul>

After students construct the line segment they create opportunities to prove congruency using a number of different ways. They must provide clear explanations for their construction and take appropriate measures, (use of Axiom 4 or alternative methods) to validate their conclusions.

This opportunity to play with the construction and provide alternative proofs should encourage students to observe geometrical consequences for each action taken (see task 2). The particular question should help students appreciate the power of axiom 4 when proving geometrical statements, this in turn should provide the opportunity for them to enhance their geometrical thinking and logical reasoning.

<p>Constructions:</p> <ol style="list-style-type: none"> <li>Bisector of a given angle, using only compass and straight edge.</li> <li>Perpendicular bisector of a segment, using only compass and straight edge.</li> <li><b>Line perpendicular to a given line <math>l</math>, passing through a given point not on <math>l</math>.</b></li> <li>Line perpendicular to a given line <math>l</math>, passing through a given point on <math>l</math>.</li> <li>Line parallel to a given line, through a given point.</li> <li>Division of a line segment into 2 or 3 equal segments, without measuring it.</li> <li><b>Division of a line segment into any number of equal segments, without measuring it.</b></li> <li>Line segment of a given length on a given ray.</li> <li>Angle of a given number of degrees with a given ray as one arm.</li> <li>Triangle, given lengths of three sides</li> <li>Triangle, given SAS data</li> <li>Triangle, given ASA data</li> <li>Right-angled triangle, given the length of the hypotenuse and one other side.</li> <li>Right-angled triangle, given one side and one of the acute angles (several cases).</li> <li>Rectangle, given side lengths.</li> </ol>	– complete the constructions specified
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## ABOUT THE UNIT AND THE LESSON

Extract from the Syllabus

Topic Specific: Strand 2: Geometry and Trigonometry(page 18)

A student should be able to:

- construct a variety of geometric shapes and establish their specific properties or characteristics
- solve geometrical problems and in some cases present logical proofs
- analyse and process information presented in unfamiliar contexts.

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
<b>2.1 Synthetic geometry</b>	<p>Concepts (see <i>Geometry Course</i> section 9.1 for OL and <b>10.1 for HL</b>)</p> <p>Axioms (see <i>Geometry Course</i> section 9.3 for OL and <b>10.3 for HL</b>):</p> <ol style="list-style-type: none"> <li>[Two points axiom] There is exactly one line through any two given points.</li> <li>[Ruler axiom] The properties of the distance between points</li> <li>[Protractor Axiom] The properties of the degree measure of an angle</li> <li>Congruent triangles (SAS, ASA and SSS)</li> <li>[Axiom of Parallels] Given any line <math>l</math> and a point <math>P</math>, there is exactly one line through <math>P</math> that is parallel to <math>l</math>.</li> </ol> <p>Theorems: [Formal proofs are not examinable at OL. <b>Formal proofs of theorems 4, 6, 9, 14 and 19 are examinable at HL.</b>]</p> <ol style="list-style-type: none"> <li>Vertically opposite angles are equal in measure.</li> <li>In an isosceles triangle the angles opposite the equal sides are equal. Conversely, if two angles are equal, then the triangle is isosceles.</li> <li>If a transversal makes equal alternate angles on two lines then the lines are parallel, (and converse).</li> <li>The angles in any triangle add to <math>180^\circ</math>.</li> <li>Two lines are parallel if and only if, for any transversal, the corresponding angles are equal.</li> <li>Each exterior angle of a triangle is equal to the sum of the interior opposite angles.</li> <li>In a parallelogram, opposite sides are equal and opposite angles are equal (and converses).</li> <li>The diagonals of a parallelogram bisect each other.</li> <li><b>If three parallel lines cut off equal segments on some transversal line, then they will cut off equal segments on any other transversal.</b></li> <li><b>Let <math>ABC</math> be a triangle. If a line <math>l</math> is parallel to <math>BC</math> and cuts <math>[AB]</math> in the ratio <math>s:t</math>, then it also cuts <math>[AC]</math> in the same ratio (and converse).</b></li> <li>If two triangles are similar, then their sides are proportional, in order (and converse).</li> <li>[Theorem of Pythagoras] In a right-angled triangle the square of the hypotenuse is the sum of the squares of the other two sides.</li> </ol>	<ul style="list-style-type: none"> <li>– recall the axioms and use them in the solution of problems</li> <li>– <b>use the terms: theorem, proof, axiom, corollary, converse and implies</b></li> <li>– apply the results of all theorems, converses and corollaries to solve problems</li> <li>– <b>prove the specified theorems</b></li> </ul>

**Lesson Idea 3.15**

**Title**

Revision of second year material – Lesson Ideas 2.27 -2.34

**Resources**

*Online resources on the Project Maths website*

A mathematical instruments set, Dynamic software package

*Junior Certificate Guidelines for Teachers* (DES 2002, Government Publications Sales Office). It is also available to download at [www.projectmaths.ie](http://www.projectmaths.ie).

**Content**

These lessons will involve the students in investigating and understanding:

- **The use of the term: theorem, proof, axiom, corollary and implies**
- Axioms 1, 2, 3 and 5
- Constructions 8, 9, 5, 6, 10, 11, 12, 1, 2, 3, 4 and (7 for higher level)
- Theorems 1 – 6
- **Proof of Theorems 4 and 6**
- Axiom 4 : Congruent triangles (SSS, ASA, SAS)
- Alternate angles and corresponding angles

**Lesson Idea 3.16**

**Title**

Revision of second year material – Lessons 2.35 -2.39

**Resources**

*Online resources on the Project Maths website*

A mathematical instruments set, Dynamic software package

*Junior Certificate Guidelines for Teachers* (DES 2002, Government Publications Sales Office). It is also available to download at [www.projectmaths.ie](http://www.projectmaths.ie).

**Content**

These lessons will involve the students in investigating and understanding:

- Translations, axial symmetry, central symmetry and rotations
- Properties of parallelograms
- Square, rhombus, parallelogram, rectangle
- Theorems 9,10
- **Proof of Theorem 9**

**Section 3 : Synthetic Geometry 1**

**While proofs are not the issue as regards informal introduction, it is important that students are kept aware that the theorems build logically.**

Concepts relevant to this section in synthetic geometry:  
 Set, plane, point, line, ray, angle, real number, length, degree. Triangle, right-angle, congruent triangles, parallel lines, area, line segment, collinear points, distance, reflex angle, ordinary angle, straight angle, null angle, full angle, supplementary angles, vertically-opposite angles, acute angle, obtuse angle, angle bisector, perpendicular lines, perpendicular bisector of a line segment, isosceles triangle, equilateral triangle, scalene triangle, right-angled triangle, exterior angles of a triangle, interior opposite angles, alternate angles, corresponding angles, transversal line, circle.

This is a suggested sequence for teaching Third Year students. It includes the material in the Common Introductory Course and also deals with some triangle constructions and congruence of triangles. Refer to the syllabus for the "Geometry for Post - primary School Mathematics" which sets out the agreed course in geometry for both Junior Certificate Mathematics and Leaving Certificate Mathematics. Strand 2 of the relevant syllabus document specifies the learning outcomes at the different syllabus levels.

**Refer to Appendix A: "Geometry - Thinking at different levels - The Van Hiele Theory"**

**Refer to Appendix B for the "Guide to Axioms, Theorems and Constructions for all Levels".** In Appendix B, \* indicates that proof of the relevant theorem is required for JCHL and LCHL and \*\* indicates that proof of the relevant theorem is required for LCHL only.

Teachers are also strongly recommended to use the Geometry Lesson Idea ideas in the "Junior Certificate Guidelines for Teachers" (DES 2002, Government Publications Sales Office). It is also available to download at [www.projectmaths.ie](http://www.projectmaths.ie).

As outlined at the workshops, the use of manipulative products such as "geostrips", "angles", geo-boards etc. can make the learning so much more enjoyable for students of all perceived abilities.

The first 2 lesson ideas for Geometry of year 3 are designed to give the students a chance to revisit the material met in first and second year Geometry. It is recommended that new activities and challenges be introduced during this revision so that students do not see it as too much repetition and that they can see new ways of investigating familiar situations.

**Note on experimentation and experimental results:**

With experimentation, when we measure, the results are only approximations and won't agree exactly. It is important for students to report faithfully what they find e.g. for a triangle they could find that the sum of the angles to be 179° or 181 etc. The conclusion is that the angles appear to add up to 180°. This is a plausible working assumption. There is a distinction between what you can discover and what you can prove.

In first year we were experimenting and using words. In second year we start to problem solve in concrete situations and then in third year we prove things.

See Section 8.2 (From Discovery to Proof) of *Geometry for Post - primary School Mathematics*.

**Lesson Idea 2.32**

**Title**

Revision of first year material – a complete recap on Lesson Ideas 1.27 and 1.28

**Resources**

*Online resources on the Project Maths website*

A mathematical instruments set

Dynamic software package

*Syllabus: Geometry Course for Post-Primary School Mathematics*

**Content**

These lessons will involve the students in investigating and understanding:

- **Construction 12:** Triangle given ASA - Congruent triangles (Axiom 4)
- More constructions of triangles with SSS, SAS and ASA
- By construction, show that AAA and AAS are not sufficient conditions for congruence.
- **Theorem 2:** (i) In an isosceles triangle the angles opposite the equal sides are equal.  
 (ii) Conversely, if two angles are equal, then the triangle is isosceles

**Lesson Idea 2.33**

**Title**

Revision of first year material – a complete recap on Lesson Ideas 1.29 and 1.30

**Resources**

*Online resources on the Project Maths website*

A mathematical instruments set

*Syllabus: Geometry Course for Post-Primary School Mathematics*

*Junior Certificate Guidelines for Teachers*(DES 2002, Government Publications Sales Office €3.81). It is also available to download at [www.projectmaths.ie](http://www.projectmaths.ie).

Dynamic software package

**Content**


These lessons will involve the students in investigating and understanding:

- Alternate angles by examples and measuring
- **Theorem 3:** (i) If a transversal makes equal alternate angles on two lines, then the lines are parallel.  
 (ii) Conversely, if two lines are parallel, then any transversal will make equal alternate angles with them.
- **\*Theorem 4:** The angles in any triangle add to 180°. **(Proof required for Higher Level only)**

**FLOW OF THE UNIT:**

<b>Lesson</b>		<b># of lesson periods</b>
Year 1	Prior Knowledge CIC	15
Year 3		
1	Revision of key geometrical terms from CIC	1
2	Introduction to Geometry and the Axioms 1- 3, highlighting their role	1
3	Identifying and discussing the key attributes of triangles and quadrilaterals what makes them unique.	1
4	Axiom 4 SSS, SAS Axiom 4: ASA, RHS	1
Research Lesson	The value and application of Axiom 4, using it to provide proof of solutions	1
7	Further investigation into axiom 4 and the introduction to Formal Proofs where understanding of congruency is required	1

## FLOW OF THE LESSON

Teaching Activity	Points of Consideration
<p><b>1. Introduction</b></p> <p><i>5min</i></p>	<p>This comprises of a short discussion based on prior knowledge. The topics discussed will be:</p> <ul style="list-style-type: none"> <li>➤ What is congruency, discuss definition.</li> <li>➤ What do they know about congruent triangles</li> <li>➤ What do they know about rectangles.....What makes them unique/similar to other quadrilaterals. ie. properties</li> </ul>
<p><b>2. Posing the Task</b></p> <p><i>3min</i></p> <p><i>(see Appendix 1 for Worksheet)</i></p>	<p>Students' introduction to the first task of the lesson.</p> <p>With the construction of one line segment, and the use of pencil, ruler and/or scissors. Prove the triangles you have created are congruent in as many ways as possible.</p> <p>List Congruent Triangles and state the method of proof you used.</p> <p>A <span style="margin-left: 150px;">B <u>Explanation</u></span></p> <div style="text-align: center; margin-top: 10px;">  </div> <p>C <span style="margin-left: 150px;">D</span></p>

### 3. Anticipated Student Responses

(see Appendix 4 for samples of students work)

Students will be provided with a scissors a ruler, this should enable them to use their understanding of the definition of congruency to show the triangles are equal.

R1 - Cut out the two triangles and show that they fit exactly on top of each other, therefore they are equal.

R2 - With the use of the ruler a student can measure all three sides and state that the sides are all the same therefore the shape is congruent.

R3 - A Student may use the properties of rectangles, to conclude that the triangles are congruent.

Property: Opposite sides are equal in rectangle and the third side (diagonal) is common....SSS

R4 - A student may use the right angle in a rectangle along with one side and the hypotenuse(diagonal) ...RHS

R5 - A student may use the properties of parallel lines (alternate angles) and the diagonal to give us congruent triangles...ASA



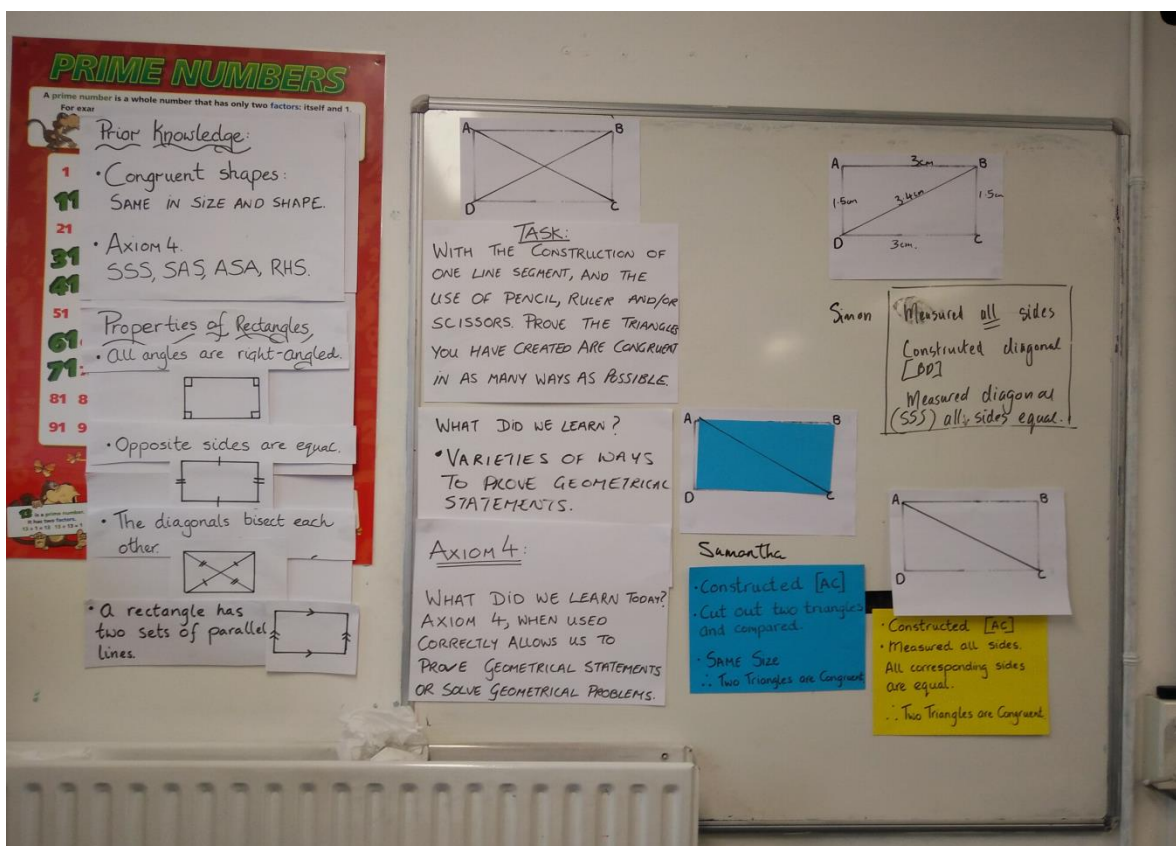
	<p>R6 - A student may use the properties of parallel lines, rectangles and the diagonal to give us congruent triangles.....SAS</p> <p>Errors may occur when students fail to acknowledge the importance of the order of information used ie SAS: angle contained between the two sides and ASA: side used contained between the two angles used.</p> <p>Note: The boardplan and possible solutions are using the diagonal [AC], all these methods are also valid using the diagonal [BD], giving us a total of 12 possible solutions</p> <p>Hints that may be given during the exercise?</p> <p>If they can not find any further methods,  <i>DID THEY USE THEIR SCISSORS?</i>  <i>HAVE THEY USED THEIR RULER?</i>  <i>HAVE THEY USED THE KNOWLEDGE ON THE BOARD?</i>      IS THAT THE ONLY LINE SEGMENT YOU CAN CONSTRUCT?</p>
<p><b>4. Comparing and Discussing</b></p> <p><i>Task time: 10 min</i></p>	<p>The board plan will be organised from the most basic (cut out triangles to compare) to the most complex where a student using the properties of both the rectangle and</p>

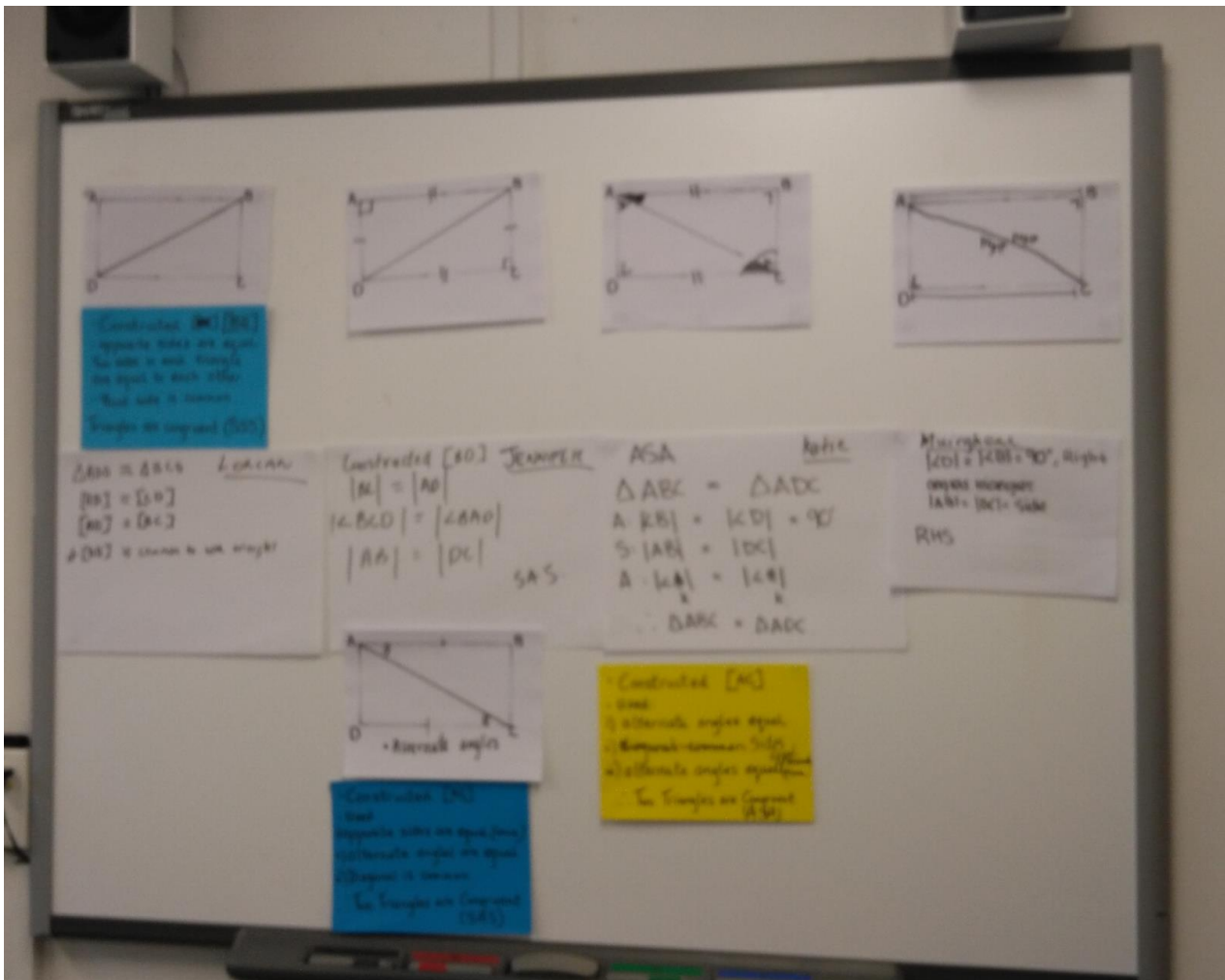
<p><i>Comparison and discussion time: 10 min</i></p>	<p>parallel lines when deducing congruency.</p> <p>The purpose is for students to appreciate the equally valid methods of proof/ value of axiom4 as an efficient way to identify if triangles are congruent</p> <p>Axiom 4, has four statements we accept as true which enable us to effectively prove congruency of triangles and in turn we may use this to deduce things about other shapes</p>
<p><b>5. Summing up</b></p> <p><i>10 min</i></p> <ul style="list-style-type: none"> <li>● <i>to include reflection 5min</i></li> <li>● <i>discussion of the homework 5min</i></li> <li>● <i>handout Homework Sheet</i></li> </ul>	<p>What did we learn today?</p> <p>Given that we are studying Euclidean Geometry which methods should we try to use when proving triangles are congruent?</p> <p>If I was to pose the same task with an extra condition.</p> <p><b>With the construction of one line segment, and the use of pencil, ruler and/or scissors. Prove the triangles you have created are congruent in as many ways as possible. You may not use any of the sides of the rectangles in your proof. Can I still use all the methods on the board?</b></p> <p><b>Inform students that the boardplan will be put up on edmodo to assist them in helping them with the second task, they can complete the same question with restriction.</b></p>

## EVALUATION

- Teachers involved with the lesson plan development, along with RDO for Project Maths developed a student observation record( Appendix 3)
- Teachers involved identified who will be observed, what will be observed, how to record data, etc.
- Each teacher will be assigned a row of 6 - 8 students, they will observe the interaction, record questions and the work of the students by taking pictures, etc.
- Teachers will take written notes and snapshots of the work being done at different stages of the class, they will take note of any observations related to the lesson plan for the post lesson discussion?
- Teachers should be looking to see if there is an improvement around the understanding of Axiom 4, do the students engage in the task set, do they appreciate other solutions offered by their classmates. Does the board plan allow student understanding to develop?
- The aim for teachers is to try to collate as much evidence as possible in this regard, they should observe their students work and engagement during the whole class discussion. Did the students they were assigned attain the learning goals set?,etc.

## BOARD PLAN (SEE APPENDIX 2 FOR PLANNING STAGE)





**POST-LESSON REFLECTION**

Learning Goals were achieved, however there was a distinct pattern observed.

SSS – Most popular method

RHS – Very popular method

ASA – Only 2 students attempted this-only one successfully. They did come up with an alternative ASA solution using the right angle.

- Alternate Angles – Students had some difficulty explaining the term.
- Some confusion between corresponding and alternate angles was identified and will be addressed during next lesson.
- Cutting out to show congruency is an excellent idea.
- The multiple approaches strategy proved highly successful, enabling students to compare and analyse other answers.

- The board plan is extremely beneficial but it requires an awful lot of work and preparation.
- The observing teachers were very impressed with the effectiveness of the lesson; *“I am going to do this exact lesson with my leaving certs this evening”*

Students were:

- Highly engaged and enjoyed the challenge set.
- Given an opportunity to self-correct and encouraged to make sure they had used correct sides/angles.
- Able to provide solutions and verbalise their reason behind answers given.
- Interested in the whole class discussion, enjoyed listening to other students explanations and expressed any difficulties they had in the understanding of any explanations.

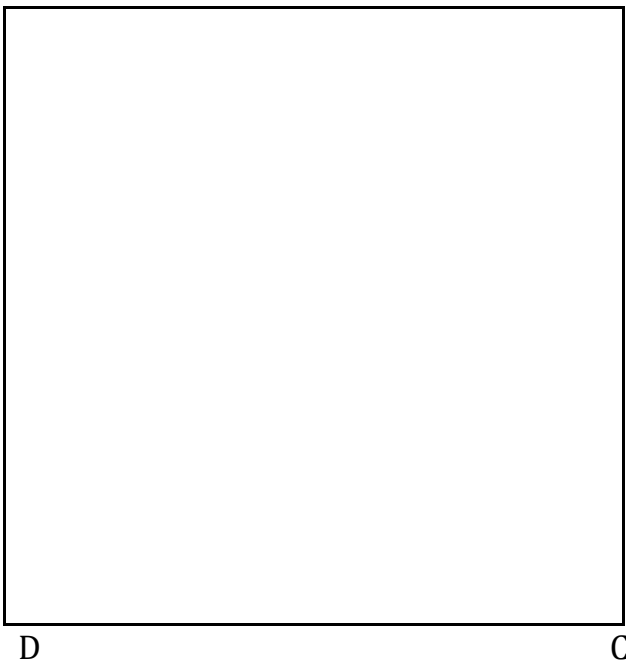
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*Appendix 1: Third Year Congruency Investigation:*

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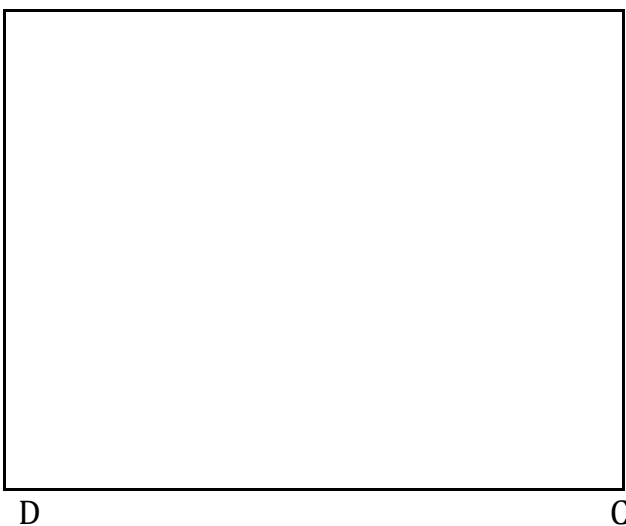
With the construction of one line segment, and the use of pencil, ruler and/or scissors, prove the triangles you have created are congruent in as many ways as possible. List Congruent Triangles and state the method of proof you used.

A B **Explanation:**



List Congruent Triangles and state the method of proof you used.

A B **Explanation:**



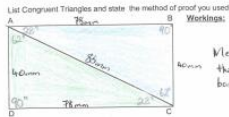
## Appendix 2: Board Plan (planning Stage)

### Cut out and compared



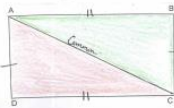
Workings:  
Construct this figure [AC]  
Cut out  $\rightarrow$  they are congruent

### Measuring sides & angles.



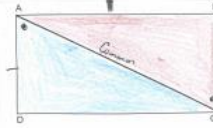
Workings:  
Measured all sides  
therefore ~~congruent~~  
based on definition.

### Compared sides only



Workings:  
Proved they congruent  
based on all sides corresponding  
using the fact that  
opposite sides in rectangle are  
equal.

### Compared sides and angle in between.



Workings:

SAS

- Property of rectangle  $\rightarrow$  Side
- Property of rectangle & diagonal  
 $\rightarrow$  alternate angles
- Common Side

### Compared angle and side in between.



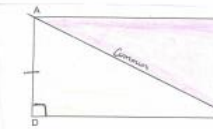
Workings:

• Constructed [AC]

- Use property of parallel lines  
"alternate angles"

ASA

### Compared sides and use right angle.



- Property of rectangle  
"angles are 90°"
- Hypotenuse - common
- Property of rectangle  
"opposite sides are equal"

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*Appendix 3: Student Observation Record*

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<b>BEGINNING OF LESSON:</b>						
<b>Observe level of difficulty with homework/previous class. If no difficulty tick the box for each student. If student has difficulty please identify issues.</b>						
	<b>Student 1</b>	<b>Student 2</b>	<b>Student 3</b>	<b>Student 4</b>	<b>Student 5</b>	<b>Student 6</b>
<b>Does the student show</b>						
<b>(i) Understanding of the term congruency</b>						
<b>(ii) Knowledge of the properties of a rectangle</b>						
<b>(iii) Knowledge of Axiom4</b>						



<b>(iv) Knowledge that properties of parallel lines can be used in rectangles</b>						
<b>Any other observations:</b>						

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**DURING LESSON:**

**Observe student interaction. If no difficulty tick the box for each student. If student has difficulty please identify issues.**

	<b>Student 1</b>	<b>Student 2</b>	<b>Student 3</b>	<b>Student 4</b>	<b>Student 5</b>	<b>Student 6</b>
Does the student have any initial difficulty with the task/construction set.						
On a scale of 1 – 5, does the student engage with the task and attempt to prove congruency in a variety of ways						
As the lesson progresses does the student stay engaged with the task.						
<b>Other observations</b>						

**LESSON CONCLUSION:**

**Observe student interaction. Check student level of confidence with simultaneous equations question.**

	<b>Student 1</b>	<b>Student 2</b>	<b>Student 3</b>	<b>Student 4</b>	<b>Student 5</b>	<b>Student 6</b>
Rate student understanding of congruency Scale 1-3 where : 1= poor 2 = some understanding 3 = competent						
Rate student understanding of use of congruency to validate proof, Scale 1-3 where : 1= poor 2 = some understanding 3 = competent						
Students can recognise, understand and respect equally valid solutions to prove congruency Scale 1-3 where : 1= poor 2 = some understanding						

3 = competent						
<i>Other observations</i>						

**LESSON CONCLUSION:**

**Observe student interaction.**

	<b>Student 1</b>	<b>Student 2</b>	<b>Student 3</b>	<b>Student 4</b>	<b>Student 5</b>	<b>Student 6</b>
<i>Other observations</i>						
<b>Notes</b>						

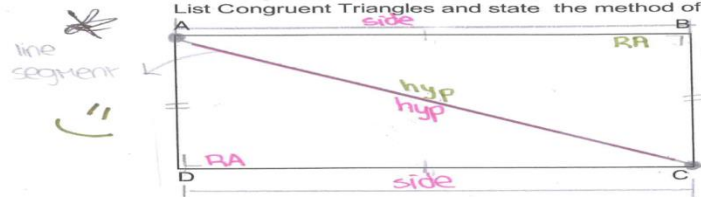
<b>Recommended changes to lesson plan</b>	
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## Appendix 4: Sample of Students Work

### Third Year Congruency Investigation:

With the construction of one line segment, and the use of pencil, ruler and/or scissors. Prove the triangles you have created are congruent in as many ways as possible.

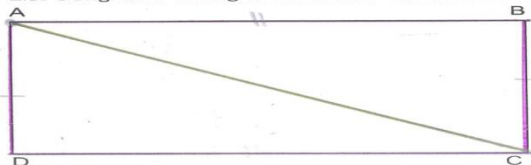
List Congruent Triangles and state the method of proof you used.



#### Explanation:

A hypotenuse & a side  
 BHS - Both triangles have a right angle  
 SSS - they both have sides (three)  
 SAS - they both have two sides and one angle  
 $\angle B = \angle D = 90^\circ$  - Right Angle  
 $|AC| = |AC|$      $|BC| = |AD|$   
 $|AB| = |DC|$

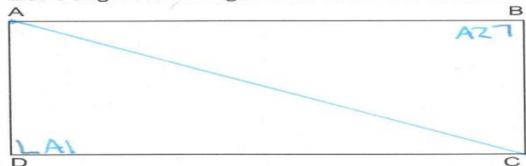
List Congruent Triangles and state the method of proof you used.



#### Explanation:

SSS - They both have three sides  
 one on each are equal  
 in measure as opposite  
 sides are equal  
 another one of each  
 side

List Congruent Triangles and state the method of proof you used.



#### Explanation:

SAS - they both have two sides  
 and one angle.

$|AB| = |DC|$   
 $|BC| = |AD|$

excellent When one triangle is flipped, they are both identical.

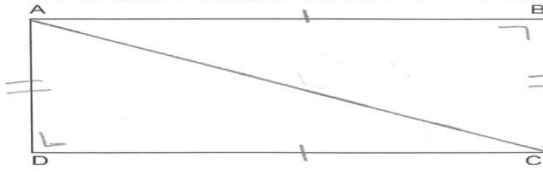
→ axial symmetry. In order for both triangles to be equal, the sides all must be the same = properties of a rect.

# AXIOM 4:

**Third Year Congruency Investigation:**

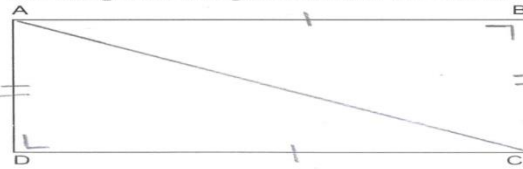
With the construction of one line segment, and the use of pencil, ruler and/or scissors. Prove the triangles you have created are congruent in as many ways as possible.

List Congruent Triangles and state the method of proof you used.



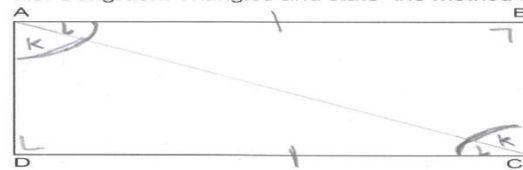
**Explanation:** SSS  
 $\triangle ABC = \triangle ADC$   
 S:  $|AB| = |DC|$   
 S:  $|BC| = |AD|$   
 S:  $|AC| = |AC|$

List Congruent Triangles and state the method of proof you used.



**Explanation:** SAS  
 $\triangle ABC = \triangle ADC$   
 S:  $|AB| = |DC|$   
 A:  $\angle B = \angle D$   
 S:  $|BC| = |AD|$

List Congruent Triangles and state the method of proof you used.



**Explanation:** ASA  
 $\triangle ABC = \triangle ADC$   
 A:  $\angle B = \angle D$   
 S:  $|AB| = |DC|$   
 A:  $\angle K = \angle L = 90^\circ$   
 $\angle K + \angle L = \angle K + \angle L$   
 $\therefore \angle L = \angle L$

↳ Alternate.



**Third Year Congruency Investigation:**

With the construction of one line segment, and the use of pencil, ruler and/or scissors. Prove the triangles you have created are congruent in as many ways as possible.

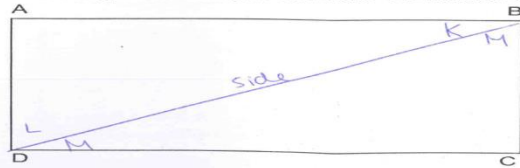
List Congruent Triangles and state the method of proof you used.



**Explanation:**

*Symmetrical*

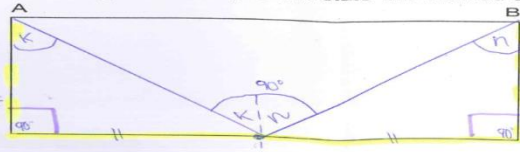
List Congruent Triangles and state the method of proof:



**Ex:**

$\angle K = \angle M$   
 hypotenuse is common  
 $\angle L = \angle N$  (alternate angles)  
ASA

List Congruent Triangles and state the method of proof you used.



**Explanation:**

$\angle K = \angle N$  \* - work needed for this conclusion  
 $\angle D = \angle C$  ( $90^\circ$ ) - A  
 $|AD| = |BC|$  ... S opposite side

(SAS)

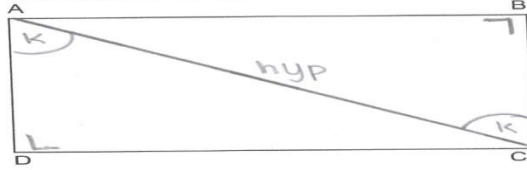
$|AD| = |BC|$  ... S  
 $|DC| = |BC|$  ... M  
 $\angle ADC = \angle BCE$

$\triangle ABC$  is  
 congruent to  
 $\triangle DEF \Rightarrow$   
 $\triangle ABC \cong \triangle DEF$

**Third Year Congruency Homework**

1. With the construction of one line segment, and the use of pencil, ruler and/or scissors. Prove the triangles you have created are congruent without using the sides of the rectangle in your proof.

List Congruent Triangles and state the method of proof you used.



**Explanation:**

$\Delta ABC = \Delta ADC$  **ASA**

A:  $|\angle B| = |\angle D|$

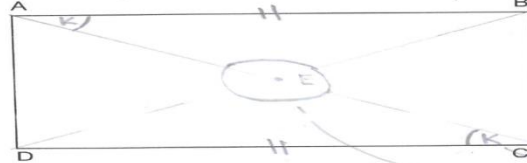
S:  $|AC| = |AC|$

A:  $|\angle K| = |\angle K| \Rightarrow$  alternate angles.

*excellent*

2. With the construction of two line segments Prove that two of the triangles you have drawn are congruent, using axiom 4.

List Congruent Triangles and state the method of proof you used.



**Explanation:**

$\Delta AEB = \Delta DEC$

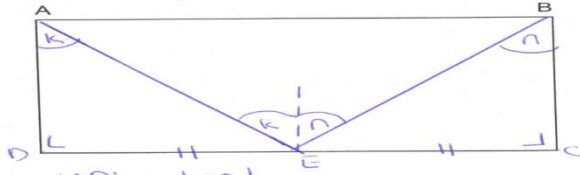
A:  $|\angle K| = |\angle K| \Rightarrow$  alternate

S:  $|AB| = |DC|$

A:  $|\angle AEB| = |\angle CED|$

transversal

Extra Workings:



**Explanation:**

$\Rightarrow a + b = 180^\circ$   
 $b + c = 180^\circ$

$\Delta AEB \cong \Delta DEC$  (ASA)

$|AD| = |BC|$   
 $|\angle D| = |\angle C|$   
 $|DE| = |CE|$

$\therefore ADE \cong DCE$  [SAS]

$\Rightarrow$

$\angle K = \angle n$   
 $\angle D = \angle C$   
 $|AD| = |BC|$