For the lesson on 1st March 2016
At Midleton College, Form 2 class
Teacher: Mitch McGuckin

Lesson plan developed by: [Catriona Hassey, Mitch McGuckin, Laura Guest, Margaret Rumley, Michael Moynihan and Celeste Quinlan]

1. **Title of the Lesson:**
   A fixed perimeter does not imply a fixed area.

2. **Brief description of the lesson**
   Given a wire of length 20m, explore the various dimensions and areas of rectangular paddocks that can be made using all of the wire. The area is to be maximised for the paddocks usefulness e.g. storing livestock etc.

3. **Aims of the Lesson:**

   Long Term
   a) I'd like my students to appreciate that mathematics can be used to solve real world problems and the meaningfulness of mathematics in everyday situations.
   b) I'd like to emphasise to students that a problem can have several equally valid solutions.
   c) I'd like my students to be able to identify patterns in the results they achieve.
   d) I'd like my students to be able to use the pattern they have found to extrapolate and apply the same logic to related situations and thereby become independent thinkers and learners.

   Short Term
   a) Students will understand that a fixed perimeter does not imply a fixed area.
   b) Students will discover that a maximum area, for any rectangular shape is a square.
   c) Students will be able to present their data in different formats to illustrate underlying patterns.
   d) Students will determine the type of pattern and hence derive the rule for the pattern.
   e) Students will relate the rule of the pattern to the context of the question.

4. **Learning Outcomes:**

   As a result of studying this topic, students will be able to:
   a) Develop an understanding of the relationship between the area and perimeter of a rectangular shape.
   b) Develop the ability to present the data, in tabular and graphic formats, to illustrate the pattern of findings.
   c) For students to know and understand that a fixed perimeter does not imply a fixed area.

5. **Background and Rationale**

   We recognize that students are challenged by spatial reasoning and particularly by geometry problems in an unusual context.
   We recognize that students have difficulties noticing patterns and using algebra to generalize and that these techniques can be applied to problems that initially seem unconnected.

6. Research

We researched Irish students' performance in state exam questions and also in recent PISA studies and found that Irish students do not perform as well in "space and shape" problems.
We decided to look for a problem involving geometry that is suitable for junior cert maths students.
One group member suggested that we look at the different areas that can be formed from a set perimeter

7. About the Unit and the Lesson

This lesson spans a few units; namely algebra, patterns, functions and geometry.
This second year class group worked on area of rectangles in first year, number patterns in first term of second year and should be familiar with creating tables, graphs and deriving the general rule for linear and quadratic patterns. They have also covered the concept of a function.

8. About the Unit and the Lesson

Strand 3 : Number   Page 23 from the Junior cert Syllabus Handbook

Topic	**Description of topic** Students learn about	**Learning outcomes** Students should be able to
3.4 Applied measure	Measure and time. 2D shapes and 3D solids, including nets of solids (two-dimensional representations of three-dimensional objects). Using nets to analyse figures and to distinguish between surface area and volume. Problems involving perimeter, surface area and volume. Modelling real-world situations and solve a variety of problems (including multi-step problems) involving surface areas, and volumes of cylinders **and prisms.** The circle and develop an understanding of the relationship between its circumference, diameter and π.	– calculate, interpret and apply units of measure and time – solve problems that involve calculating average speed, distance and time – investigate the nets of rectangular solids – find the volume of rectangular solids and cylinders – find the surface area of rectangular solids – identify the necessary information to solve a problem – select and use suitable strategies to find length of the perimeter and the area of the following plane figures: disc, triangle, rectangle, square, and figures made from combinations of these – draw and interpret scaled diagrams **– investigate nets of prisms (polygonal bases) cylinders and cones** **– solve problems involving surface area of triangular base prisms (right angle, isosceles, equilateral), cylinders and cones** **– solve problems involving curved surface area of cylinders, cones and spheres** **– perform calculations to solve problems involving the volume of rectangular solids, cylinders, cones, triangular base prisms (right angle, isosceles, equilateral), spheres and combinations of these**

Strand 4:Algebra   Page 27 Junior Cert Syllabus Handbook

Topic	**Description of topic** Students learn about	**Learning outcomes** Students should be able to
4.1 Generating arithmetic expressions from repeating patterns	Patterns and the rules that govern them; students construct an understanding of a relationship as that which involves a set of inputs, a set of outputs and a correspondence from each input to each output.	– use tables to represent a repeating-pattern situation – generalise and explain patterns and relationships in words and numbers – write arithmetic expressions for particular terms in a sequence
4.2 Representing situations with tables, diagrams and graphs	Relations derived from some kind of context – familiar, everyday situations, imaginary contexts or arrangements of tiles or blocks. Students look at various patterns and make predictions about what comes next.	– use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations (exponential relations limited to doubling and tripling) – develop and use their own generalising strategies and ideas and consider those of others – present and interpret solutions, explaining and justifying methods, inferences and reasoning
4.3 Finding formulae	Ways to express a general relationship arising from a pattern or context.	– find the underlying formula written in words from which the data are derived (linear relations) **– find the underlying formula algebraically from which the data are derived (linear, quadratic relations)**
4.4 Examining algebraic relationships	Features of a relationship and how these features appear in the different representations. Constant rate of change: linear relationships. Non-constant rate of change: quadratic relationships. Proportional relationships.	– show that relations have features that can be represented in a variety of ways – distinguish those features that are especially useful to identify and point out how those features appear in different representations: in tables, graphs, physical models, and formulas expressed in words, and algebraically – use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others – recognise that a distinguishing feature of quadratic relations is the way change varies – discuss rate of change and the y-intercept; consider how these relate to the context from which the relationship is derived, and identify how they can appear in a table, in a graph and in a formula – decide if two linear relations have a common value – investigate relations of the form $y=mx$ and $y=mx+c$ – recognise problems involving direct proportion and identify the necessary information to solve them

Strand 5: Functions   Page 31 Junior Cert Handbook

Topic	Description of topic Students learn about	Learning outcomes Students should be able to
5.1 Functions	The meaning and notation associated with functions.	– engage with the concept of a function, domain, co-domain and range – make use of function notation $f(x) =$, $f: x \rightarrow$, and $y =$
5.2 Graphing functions	Interpreting and representing linear, quadratic and exponential functions in graphical form.	– interpret simple graphs – plot points and lines – draw graphs of the following functions and interpret equations of the form $f(x) = g(x)$ as a comparison of functions • $f(x) = ax + b$, where $a, b \in \mathbf{Z}$ • $f(x) = ax^2 + bx + c$, where $a \in \mathbf{N}$; $b, c \in \mathbf{Z}$; $x \in \mathbf{R}$ • **$f(x) = ax^2 + bx + c$, where $a, b, c \in \mathbf{Z}$, $x \in \mathbf{R}$** • **$f(x) = a2^x$ and $f(x) = a3^x$, where $a \in \mathbf{N}$, $x \in \mathbf{R}$** – use graphical methods to find approximate solutions where $f(x) = g(x)$ and interpret the results – **find maximum and minimum values of quadratic functions from a graph** – **interpret inequalities of the form $f(x) \leq g(x)$ as a comparison of functions of the above form; use graphical methods to find approximate solution sets of these inequalities and interpret the results** – graph solution sets on the number line for linear inequalities in one variable

Students learn about	*Students should be able to*
5.3 Synthesis and problem-solving skills	– explore patterns and formulate conjectures – explain findings – justify conclusions – communicate mathematics verbally and in written form – apply their knowledge and skills to solve problems in familiar and unfamiliar contexts – analyse information presented verbally and translate it into mathematical form – devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Section 2 Strand 4
Lesson Idea 2.2

Title
Representing and examining linear and quadratic relations using tables, diagrams, graphs and formulae
Content
These lessons will involve the students in investigating and understanding:

- The use of tables, diagrams, physical models, graphs and formulae as tools for representing and analysing <u>linear</u> patterns and relations in order to
    - Identify variables and constants and point out how they appear in the different representations
    - Use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others
    - Discuss rate of change and the y - intercept
    - Generalise and explain patterns and relationships in words and numbers
    - Recognise that the distinguishing of a linear relationship is a constant rate of change
    - The concept of a quadratic function as a relationship between a set of inputs and a set of outputs where each input is related to only one output
- The use of tables, diagrams, graphs and formulae as tools for representing and analysing <u>quadratic</u> patterns and relations in order to:
    - Recognise that a distinguishing feature of quadratic relations is the way change varies i.e. the rate of change of the rate of change is constant
    - The concept of a quadratic function as a relationship between a set of inputs and a set of outputs where each input is related to only one output

Lesson Idea 2.4

Title
Applied measure 2

Content
These lessons will involve the students in investigating and understanding:

- Problems involving perimeter and area of the following plane figures: disc, triangle, rectangle, square and figures made from combinations of these (revision and extension from first year)

9. Flow of the Unit:

Lesson		# of lesson periods
	Linear and Quadratic Patterns	10 × 40 mins
	Perimeter and Area Lesson	Target Lesson 1 ×50 mins
	Functions	5 × 40 mins

10. Flow of the Lesson:

Teaching Activity	Points of Consideration
1. Introduction Play video clip (first 40 seconds only) https://www.youtube.com/watch?v=GY0gYeuGE7M	Review calculation of area and perimeter of rectangular shapes.
2. Posing the Question Place *key question* on board: "Farmer Joe has 20 metres of wire fencing, show different rectangular areas he could make, using all of the available wire each time." Ask students to work individually. **(5 minutes)**	Check for understanding of question with students. Students are provided with dotted grid paper (1cm), graph paper with axes, blank sheets, 20cm of wire and should have rulers and a pencil.
3. Anticipated Student Responses. (10 minutes) R1: Student draws one or two different options, but does not record dimensions or areas produced. R2: Student draws different options and records dimensions and areas produced. R3: Student works through all options and presents them in order of size.	Check to see all students understand the question. Some may create rectangles with areas of $20m^2$. Some students may produce a limited number of rectangles, ask if they have ALL solutions.
4. Comparing and Discussing (10 minutes) Students show work on board, work from the less sophisticated to more sophisticated approach. Ask students what conclusions they can draw from the first task. R1: We have found all the areas that could be formed by natural numbers with a perimeter of 20m. R2: As the height changed, the area changed. R3: For each amount the height increased the width must decrease by the same amount. R4: Height and width must have a sum of 10m. R5: Same perimeter, different areas R6: Square gave the largest area	Prompt for conclusions, were all possible shapes drawn, if not, why did students stop at 5×5 square. Can students come up with 2 key conclusions: • Fixed perimeter does not mean fixed area • Maximum area for rectangular figures is a square Some conclusions may be useful for later tasks and discussions.

5. Extending the Task (10 minutes)

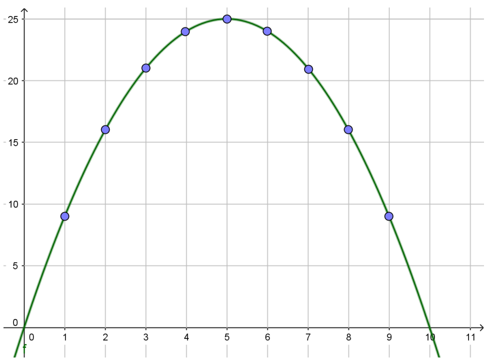
Ask the students to record their work in another way.

R1: Student produces a table of heights and related areas but omits some options

R2: Student produces a full table showing heights, widths and related areas

Height(m)	**Width(m)**	**Area(m^2)**
1	9	9
2	8	16
3	7	21
4	6	24
5	**5**	**25**
6	4	24
7	3	21
8	2	16
9	1	9

R3: Student graph their results



R4: Student makes an algebraic representation

Area = $x(10 - x) = -x^2 + 10x$

Prompt students to use height and area as variables for tables.

Have students shown all possible variables on table (i.e. all possible natural values for height, from 1 to 9)

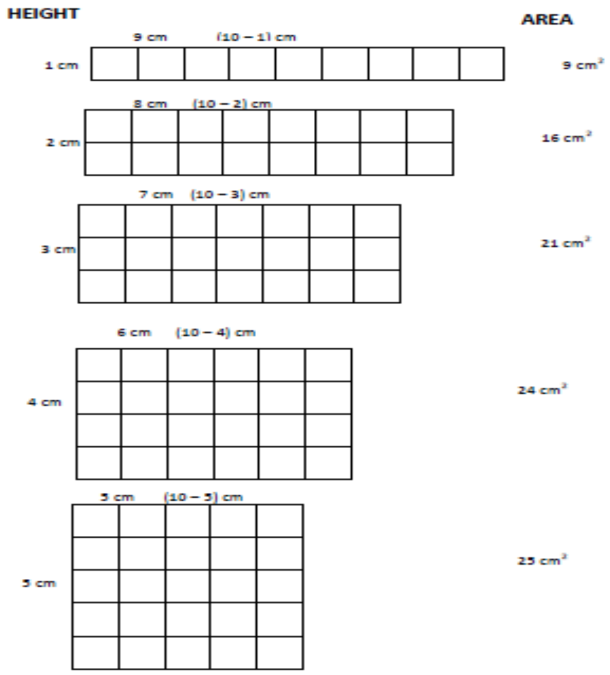
If students produce a table, ask them to show another method to represent the data.

6. Comparing and Discussing (10 minutes)	
Students show work on board, work from the less sophisticated to more sophisticated approach. Get students to do the following in order: ● Prove that the areas have a quadratic pattern. R1: Not linear (1st difference is not constant), not exponential (not a multiplication). R2: 2nd difference is constant ● Derive the rule for the quadratic pattern. R1: 2nd difference is decreasing by 2 R2: $T_n = n^2$ R3: $T_n = an^2 + bn + c$ R4: $2a= -2$, therefore $a= -1$ R5: If $T_1 = 9$, then $T_0 = 0$ (24, 21, 16, 9, **0**) therefore $c = 0$ R6: $T_n = - 1n^2 + 10n$ ● Relate the rule to the context of the original question. R1: Adjacent sides of the rectangle added to 10 R2: If one side is n the other side must be $10 - n$. Therefore the area is $n(10-n)$. ● For homework plot the function on the supplied graph sheet.	Can students see a pattern in their findings? What sort of pattern has been formed by the areas? Can students justify that the pattern of the areas is a quadratic. Do students have correct format for quadratic pattern? Have students seen connection between 2nd difference and the value of a May need to refer back to original conclusions

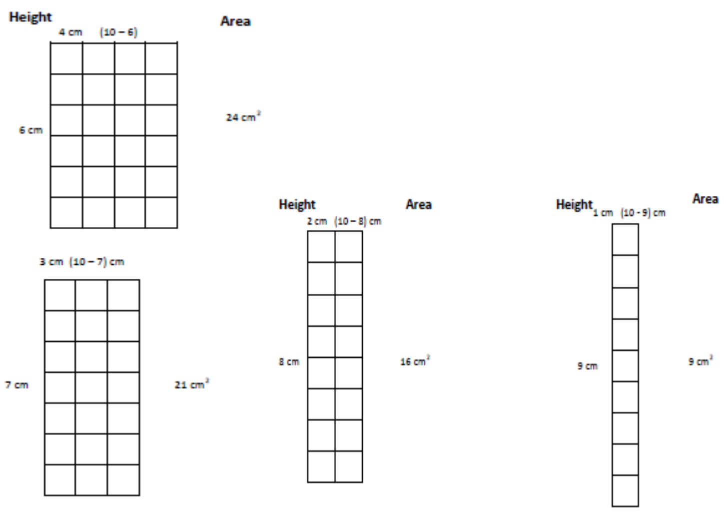
10. Evaluation

- Students will be observed by 5 observers, each to take a small group of students.
- One observer is to take photographs of both board and student work using Lesson note, others to complete student observation record for each group.
- Observation record notes: understanding demonstrated by students, work produced, engagement with the lesson and board work.
- Observers to focus on identifying areas of misunderstanding (to be brought to the attention of the class teacher) and on the approach of students to individual tasks i.e. how sophisticated their approach was.
- All student work will be collected so it can be photographed and analysed.

11. Board Plan



Farmer Joe has 20 metres of wire fencing, show different rectangular areas he could make, using all of the available wire each time.



Record and present your work in other ways.

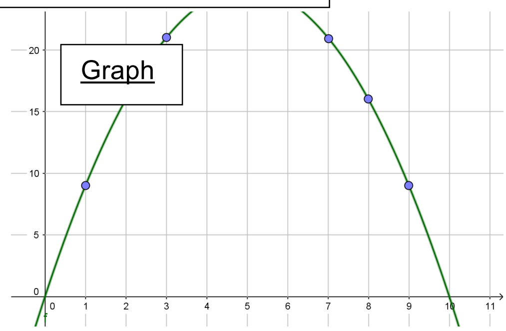
Ordered List

0, 9, 16, 21, 24, 25, 24, 21, 16, 9, 0

Table

HEIGHT cm	AREA cm^2
	9
	16
3	21
4	24
5	25
6	24
7	21
8	16
9	9

Graph



Algebra

Area = x(10 - x)

Area = $-x^2 + 10x$

where x stands for the width in metres.

Deriving the rule/formula for this quadratic pattern.

		T_1	T_2	T_3	T_4	T_5	T_6	T_7	T_8	T_9	
	0,	9,	16,	21,	24,	25,	24,	21,	16,	9,	0
1^{st} difference:		+9	+7	+5	+3	+1	-1	-3	-5	-7	-9
2^{nd} difference:			-2	-2	-2	-2	-2	-2	-2	-2	-2

$2a = -2$ → $a = -1$

$T_1 = 9$, $T_0 = 0$ → $c = 0$

$T_1 = 9 = -1(1^2) + b(1) + 0$ → $9 = -1 + b$ → $b = 10$

$\mathbf{T_n = -n^2 + 10n}$

12. Post-lesson reflection

- Some students initially misinterpreted the question and read perimeter as area, although most were quick to correct their work when asked to record their dimensions and areas. Students found it difficult to work in unfamiliar surroundings and were slightly intimidated by so many observers peering over their shoulders. As a result students were less interactive with their class teacher than normal and information had to be drawn out of them. Once students relaxed into the class they produced some fantastic and sometimes surprising work and wonderful insights.
- After the first section of the lesson, a large number of the students wrote down that "fixed perimeter can give different areas" independently of the teacher.
- As the students had recently covered the topic of patterns, they were very confident both identifying quadratic patterns and deriving the underlying function.
- The question asked was very suitable for this group of students, all students were able to access the topic at different levels. The question had relevance to both Junior Certificate and Leaving Certificate students and it was felt that the topic could provide different outcomes depending on the level required.
- The students did not use the wire which had been provided to them to model the question, therefore it would not be supplied in future. The main area for revision would be the board work, it did not flow in a coherent fashion, which made linking the tasks more difficult.
- The collaboration between different teachers provided fascinating insights into both the work that was presented and on reflection our own teaching as well. We found the idea of being "less-helpful" to be initially very difficult but seeing how the students managed encourages us to use this approach more often. While there was a significant amount of work required to produce this lesson plan, the work would certainly be easier on our next attempt. Overall we felt that is was a very worthwhile process and look forward to other groups developing these sorts of lesson plans as well, in this way we can learn from the collective experience of others, rather than reinventing the wheel each time.