

# Teaching & Learning Plans

## Arithmetic Sequences

Leaving Certificate Syllabus



# The Teaching & Learning Plans are structured as follows:



**Aims** outline what the lesson, or series of lessons, hopes to achieve.

**Prior Knowledge** points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

**Learning Outcomes** outline what a student will be able to do, know and understand having completed the topic.

**Relationship to Syllabus** refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

**Resources Required** lists the resources which will be needed in the teaching and learning of a particular topic.

**Introducing the topic** (in some plans only) outlines an approach to introducing the topic.

**Lesson Interaction** is set out under four sub-headings:

- i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.
- ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.
- iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.
- iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

**Student Activities** linked to the lesson(s) are provided at the end of each plan.

# Teaching & Learning Plan: Leaving Certificate Syllabus

## Aims

- To understand the concept of arithmetic sequences
- To use and manipulate the appropriate formulas
- To apply the knowledge of arithmetic sequences in a variety of contexts

## Prior Knowledge

Students have prior knowledge of:

- Patterns
- Basic number systems
- Sequences
- Ability to complete tables
- Basic graphs in the co-ordinate plane
- Simultaneous equations with 2 unknowns
- The  $n^{\text{th}}$  term ( $T_n$ ) of an arithmetic sequence

## Learning Outcomes

As a result of studying this topic, students will be able to:

- recognise arithmetic sequences in a variety of contexts
- recognise sequences that are not arithmetic
- apply their knowledge of arithmetic sequences in a variety of contexts
- apply the relevant formula in both theoretical and practical contexts
- calculate the value of the first term ( $a$ ), the common difference ( $d$ ) and the general term ( $T_n$ ) of an arithmetic sequence from information given about the sequence

## Catering for Learner Diversity

In class, the needs of all students, whatever their level of ability level, are equally important. In daily classroom teaching, teachers can cater for different abilities by providing students with different activities and assignments graded according to levels of difficulty so that students can work on exercises that match their progress in learning. Less able students may engage with the activities in a relatively straightforward way while the more able students should engage in more open-ended and challenging activities. In this Teaching and Learning Plan, for example teachers can provide students with different applications of arithmetic sequences and with appropriate amounts and styles of support.

In interacting with the whole class, teachers can make adjustments to suit the needs of students. **For example, the Fibonacci sequence can be presented as a more challenging topic for some students.**

Apart from whole-class teaching, teachers can utilise pair and group work to encourage peer interaction and to facilitate discussion. The use of different grouping arrangements in these lessons should help ensure that the needs of all students are met and that students are encouraged to verbalise their mathematics openly and share their learning.

## Relationship to Leaving Certificate Syllabus

Students learn about	Students working at FL should be able to	In addition, students working at OL should be able to	In addition, students working at HL should be able to
3.1 Number systems	<ul style="list-style-type: none"> <li>– appreciate that processes can generate sequences of numbers or objects</li> <li>– investigate patterns among these sequences</li> <li>– use patterns to continue the sequence</li> <li>– generate rules/formulae from those patterns</li> </ul>	<ul style="list-style-type: none"> <li>– generalise and explain patterns and relationships in algebraic form</li> <li>– recognise whether a sequence is arithmetic, geometric or neither</li> <li>– find the sum to <math>n</math> terms of an arithmetic series</li> </ul>	

Lesson Interaction			
Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
<b>Section A: To introduce arithmetic sequences or arithmetic progressions and gain an understanding of the formula <math>T_n = a + (n - 1)d</math></b>			
» Can you give me examples of patterns you have already encountered?	<ul style="list-style-type: none"> <li>• Blue, red, blue, red...</li> <li>• 2, 3, 5, 8, 12...</li> <li>• 1, 4, 9, 16, 25...</li> <li>• 3, 8, 13, 18, 23...</li> </ul>	<ul style="list-style-type: none"> <li>» Revise the concept of pattern as dealt with at Junior Certificate level.</li> </ul>	<ul style="list-style-type: none"> <li>» Are the students familiar with patterns?</li> <li>» Have students come up with examples of different patterns?</li> </ul>
» Can you now do questions 1 - 4 on <b>Section A: Student Activity 1</b> ?	<ul style="list-style-type: none"> <li>• Students should try out these questions, compare answers around the class and have a discussion about why their answers do not all agree.</li> </ul>	<ul style="list-style-type: none"> <li>» Distribute <b>Section A: Student Activity 1</b> to the students.</li> <li>» Give students time to explore and to discuss what is happening.</li> </ul>	<ul style="list-style-type: none"> <li>» Can students see that these problems have a pattern?</li> <li>» Do students recognise that any term (apart from the first term) in the various patterns is formed by adding a fixed number to the preceding term?</li> </ul>

Teaching & Learning Plan: Arithmetic Sequences

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
<p>» What have the first three questions in <b>Section A: Student Activity 1</b> got in common?</p> <p>» Sequences where you are given an initial term and where each subsequent term is found by adding a fixed number to the previous term are known as an arithmetic sequences or arithmetic progressions (APs).</p> <p>» The initial term is denoted by <math>a</math> and <math>d</math> is used to denote the common difference. <math>d</math> is the number that is added to each term to generate the next term.</p> <p>» This notation is used by mathematicians.</p>	<ul style="list-style-type: none"> <li>For each question we were given an initial value and a value that was added to each previous term to generate the next term.</li> </ul>	<p>» Write the words 'arithmetic sequence' on the board.</p> <p>» Write <math>a =</math> Initial term (First term) and <math>d =</math> number added to each consecutive term (common difference) on the board.</p>	
<p>» We call the 4<sup>th</sup> term <math>T_4</math>, the ninth term <math>T_9</math> and the <math>n^{\text{th}}</math> term <math>T_n</math>.</p> <p>» In each of the three problems on the <b>Section A: Student Activity 1</b>, ask how we got each term?</p>	<ul style="list-style-type: none"> <li>We were given an initial term <math>a</math> and the next term was found by adding <math>d</math> (the common difference) to this initial term and then the 3<sup>rd</sup> term was found by adding <math>d</math> to the second term.</li> </ul>	<p>» Write</p> <ul style="list-style-type: none"> <li>4<sup>th</sup> term <math>= T_4</math></li> <li>9<sup>th</sup> term <math>= T_9</math></li> <li><math>n^{\text{th}}</math> term <math>= T_n</math> on the board.</li> </ul>	

## Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
<ul style="list-style-type: none"> <li>» Now let's see if we can come up with a formula for <math>T_n</math>?</li> <li>» Start with <math>T_1</math>. What is <math>T_1</math> in terms of <math>a</math> and <math>d</math>?</li> <li>» Then move on to <math>T_2</math>. If we know what <math>T_1</math> is, how do we find <math>T_2</math>?</li> <li>» Write out the next three terms.</li> <li>» Do you notice a pattern occurring?</li> <li>» So what is <math>T_8</math>?</li> <li>» What is <math>T_{38}</math>?</li> <li>» What is <math>T_{107}</math>?</li> <li>» What is <math>T_n</math>?</li> </ul>	<ul style="list-style-type: none"> <li>• Students should try this themselves and compare answers around the class and have a discussion about why the answers do not all agree.</li> <li>• <math>T_1 = a</math></li> <li>• We add '<math>d</math>' to <math>T_1</math> to get <math>T_2</math> so <math>T_2 = a + d</math></li> <li>• <math>T_3 = T_2 + d = a + d + d = a + 2d</math> <math>T_4 = T_3 + d = a + 2d + d = a + 3d</math> <math>T_5 = T_4 + d = a + 3d + d = a + 4d</math></li> <li>• Yes, the number of <math>d</math>s is always one less than the term we are looking for.</li> <li>• <math>T_8 = a + 7d</math></li> <li>• <math>T_{38} = a + 37d</math></li> <li>• <math>T_{107} = a + 106d</math></li> <li>• <math>T_n = a + (n - 1)d</math></li> </ul>	<ul style="list-style-type: none"> <li>» Give students time to explore and discuss what is happening.</li> <li>» Write the following on the board as students come up with the terms. <math>T_1 = a</math> <math>T_2 = a + d</math> <math>T_3 = T_2 + d = a + d + d = a + 2d</math> <math>T_4 = T_3 + d = a + 2d + d = a + 3d</math> <math>T_5 = T_4 + d = a + 3d + d = a + 4d</math> ... <math>T_n = a + (n - 1)d</math></li> </ul> <p><b>Note:</b> Explain that this formula only applies to an arithmetic sequence that <math>d</math> always has to be a constant and <math>a</math> is the first term.</p>	<ul style="list-style-type: none"> <li>» Were students able to come up with the first 5 terms?</li> <li>» Do students recognise that for a particular sequence, <math>d</math> always has the same value?</li> <li>» Do students recognise that the 8<sup>th</sup> term is <math>T_8 = a + (8 - 1)d</math>, <math>T_{38} = a + (38 - 1)d</math>, <math>T_{107} = a + (106)d</math>, before getting <math>T_n</math>?</li> <li>» Do students understand the meanings of <math>a</math>, <math>d</math> and <math>T_n = a + (n - 1)d</math>?</li> </ul>

Teaching & Learning Plan: Arithmetic Sequences

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
<p>» Continue with <b>Section A: Student Activity 1.</b></p>		<p><b>Note:</b> Students will need to understand that <math>a</math> and <math>d</math> may be fractions, decimals or negative numbers. They should be allowed to discover this rather than being told at the beginning of the activity sheet. They should also recognise that <math>n</math> is always a positive integer.</p> <p>» Questions on <b>Section A: Student Activity 1</b> can be done incrementally and answers discussed with the class.</p>	<p>» Are students capable of answering the questions on <b>Section A: Student Activity 1</b>?</p>
<p><b>Reflection:</b></p> <p>» What is an arithmetic sequence?</p> <p>» What is <math>a</math>?</p> <p>» What is <math>d</math>?</p> <p>» What is <math>n</math>?</p> <p>» What is the meaning of <math>T_n</math>?</p> <p>» What is the formula for <math>T_n</math>?</p> <p>» For homework, complete the following questions (pick suitable questions) from the <b>Section A: Student Activity 1.</b></p>	<ul style="list-style-type: none"> <li>• An arithmetic sequence is one in which each term, apart from the first term, is generated by adding a fixed number to the preceding term.</li> <li>• <math>a</math> is the first term</li> <li>• <math>d</math> is the common difference</li> <li>• <math>n</math> is the number of terms</li> <li>• <math>T_n</math> is the <math>n^{\text{th}}</math> term</li> <li>• <math>T_n = a + (n - 1) d</math></li> </ul>	<p>» Conduct a discussion on what has been learned to date on arithmetic sequences.</p> <p>» More time may have to be spent on the questions in <b>Section A: Student Activity 1</b> should the rate of student progress require it.</p> <p>» Having cognisance of the students' abilities, select which homework questions on this activity sheet are to be completed.</p>	<p>» Are students familiar with the notation <math>a, d, n</math> and <math>T_n</math>?</p> <p>» Can students use the formula <math>T_n = a + (n - 1) d</math>?</p>



Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
<b>Section B: To further explore the concepts of arithmetic sequence and attempt more difficult exercises</b>			
<p>» Class, I would like you to divide into groups of 2 (or 3) and on a sheet of paper write a list of examples / applications of where arithmetic sequences occur in everyday life. Justify why your examples are arithmetic sequences.</p>	<ul style="list-style-type: none"> <li>• Saving regular amounts.</li> <li>• Depreciation or inflation by regular amounts.</li> <li>• Spending regular amounts.</li> <li>• Heights of buildings where each floor above the ground floor is the same height.</li> <li>• Removing regular equal amounts from a container.</li> </ul>	<p><b>Note:</b> Students need to be able to come up with examples of arithmetic sequences other than those they already met in class. They should also be able to recognise, with their teacher's help, what makes the sequences they chose arithmetic. Add examples of sequences that are not arithmetic and discuss these also.</p> <p>» Give students time to discuss their examples.</p>	<p>» Are students capable of justifying that the sequences they chose are arithmetic?</p>
<p>» Let's look again at the formula <math>T_n = a + (n - 1) d</math>.</p> <p>» Complete <b>Section B: Student Activity 2</b>.</p>		<p>» Distribute <b>Section B: Student Activity 2</b>.</p> <p>» Select questions depending on students' progress and abilities. Include key questions 8 and 12-14.</p> <p>» Give students an opportunity to attempt questions before the solutions are demonstrated on the board.</p>	<p>» Are students capable of applying the formula <math>T_n = a + (n - 1) d</math>?</p> <p>» Are students capable of developing stories from graphs?</p>

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
<b>Section C: To enable students understand how simultaneous equations can be used to solve problems involving arithmetic sequences</b>			
<ul style="list-style-type: none"> <li>» Solve this problem: "Mark's savings pattern obeys an arithmetic sequence. On the 4<sup>th</sup> week he saved €9 and on the 6<sup>th</sup> week he saved €13.</li> <li>» Design 2 equations in terms of <math>a</math> and <math>d</math> to represent Mark's savings pattern.</li> <li>» How do we solve these equations?</li> <li>» What does <math>a = 3</math> and <math>d = 2</math> mean in the context of the question?</li> <li>» How much will he save in the 10<sup>th</sup> week?</li> </ul>	<ul style="list-style-type: none"> <li>• <math>a + 3d = 9</math> <math>a + 5d = 13</math></li> <li>• Students should come up with the concept of simultaneous equations. <math>a + 5d = 13</math> <math>a + 3d = 9</math> <math>2d = 4</math> <math>d = 2</math> <math>a + 5(2) = 13</math> <math>a = 3</math></li> <li>• Mark saved €3 in the first week and increased the amount he saved each week by €2.</li> <li>• <math>T_{10} = 3 + (10 - 1) 2</math> <math>T_{10} = 21</math></li> </ul>	<ul style="list-style-type: none"> <li>» Ask an individual student to write the solution on the board and explain what he or she is doing in each step.</li> </ul> <p><b>Note:</b> Students should already be familiar with simultaneous equations, but may need a quick revision exercise on how to solve them. It should be emphasised that simultaneous equations can be represented by letters other than <math>x</math> and <math>y</math>.</p> <ul style="list-style-type: none"> <li>» The time spent on this session will depend on the students' abilities and the understanding they already have of simultaneous equations.</li> <li>» Another example may need to be used here if students are unfamiliar with this concept.</li> </ul>	<ul style="list-style-type: none"> <li>» Can students recognise what a set of simultaneous equations are?</li> <li>» Do students recognise that the <math>a</math> and the <math>d</math> have the same value in both equations?</li> <li>» Can students solve the simultaneous equations?</li> </ul>

Teaching & Learning Plan: Arithmetic Sequences

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
<p>» Complete <b>Section C: Student Activity 3</b>.</p> <p><b>Note:</b> Pick the appropriate questions for the class, reserving some for homework and exclude numbers 10-12. At this point.</p>		<p>» Distribute <b>Section C: Student Activity 3</b>.</p> <p><b>Note:</b> For question No 7, weight loss is normally not a constant loss per month hence weight loss is not a good example of an arithmetic sequence.</p>	<p>» Are students able to establish and solve the sets of simultaneous equations that represent most of the problems in <b>Section C: Student Activity 3</b>?</p>
<p>» I would like you to work in pairs and devise your own question(s) similar to the ones in this Student Activity. Then give the questions to the group next to you so that they can solve them.</p>	<ul style="list-style-type: none"> <li>• Students exchange their equations.</li> <li>• Students offer their solutions and explain how they arrived at them.</li> </ul>	<p>» Write a variety of simultaneous equations from the students on the board together with their solutions.</p> <p>» Allow students to talk through their work so that any misconceptions become apparent and are dealt with.</p>	<p>» Are students capable of devising their own questions?</p>

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
<b>Section D: To enable students deal with problems like the following:</b>			
<ul style="list-style-type: none"> <li>» A mobile phone supplier charges €5 plus €0.10 per call.</li> <li>» What is the charge when the user makes 8 calls?</li> <li>» Which term of the sequence is this?</li> <li>» Does the cost of using this supplier form an arithmetic sequence?</li> <li>» Why do you say it forms an arithmetic sequence?</li> <li>» What value has <math>a</math> in this sequence?</li> <li>» What value has <math>d</math> in this sequence?</li> <li>» Which term in the sequence represents the 10th call?</li> <li>» This is something you need to watch out for in questions.</li> <li>» Explain why it happened in this question?</li> <li>» Now attempt questions 10 -12 in <b>Section C: Student Activity 3.</b></li> </ul>	<ul style="list-style-type: none"> <li>• €5.80</li> <li>• 9th</li> <li>• Yes</li> <li>• Because you add 0.10 for every call made, so it has the common difference.</li> <li>• 5</li> <li>• 0.10</li> <li>• 11th term</li> <li>• Because the initial term was for no calls rather than one call.</li> </ul>	<ul style="list-style-type: none"> <li>» Put the following costs on the board: €5 plus €0.10 per call.</li> <li>» Allow students to talk through their work so that any misconceptions become apparent and are dealt with.</li> </ul>	<ul style="list-style-type: none"> <li>» Can students distinguish which formula to use?</li> <li>» Do students understand why in this question the term that represents the 10<sup>th</sup> call is the 11<sup>th</sup> term?</li> </ul>

Teaching & Learning Plan: Arithmetic Sequences

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
<p><b>Section D:</b> To enable students see that not all sequences are arithmetic sequences.  <b>Note:</b> the Fibonacci sequence is not specified in the syllabus, but is a famous mathematical sequence and a class of more able students may enjoy exploring this sequence. However, at this stage in the T&amp;L teachers need to challenge their students to provide examples of some sequences that are not arithmetic.</p>			
<ul style="list-style-type: none"> <li>» Are all sequences arithmetic? Explain your reason.</li> <li>» There is a famous sequence called the Fibonacci Sequence. See <b>Section D: Student Activity 4.</b></li> <li>» Is this an arithmetic sequence?</li> <li>» Can you give examples of other sequences that are not arithmetic?</li> </ul>	<ul style="list-style-type: none"> <li>• No. To be an arithmetic sequence <math>T_n - T_{n-1}</math> must be a constant for all the terms.</li> <li>• No.</li> </ul>	<ul style="list-style-type: none"> <li>» Distribute <b>Section D: Student Activity 4.</b></li> <li><b>Note:</b> Some students may need individual help with the explanation of how the sequence develops. (See the different rabbit faces.)</li> <li>» Delay telling students that this is not an arithmetic sequence. Allow students to explore this for themselves.</li> <li><b>Note:</b> While not specifically related to this Teaching and Learning Plan, this section is important for two reasons (i) the Fibonacci Sequence is an important mathematical sequence and (ii) it is very important that students are reminded that not all sequences are arithmetic.</li> </ul>	<ul style="list-style-type: none"> <li>» Are students able to identify the characteristics of an arithmetic sequence?</li> <li>» Are students able to derive the terms of the Fibonacci Sequence?</li> <li>» Have students come up with examples of sequences that are not arithmetic?</li> </ul>

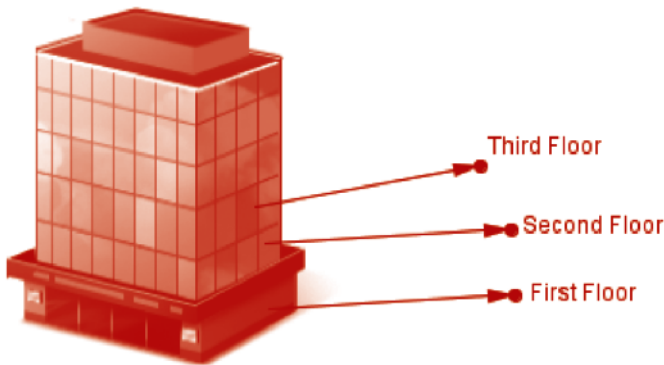
Teaching & Learning Plan: Arithmetic Sequences

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Class, complete questions 1-6 on <b>Section D: Student Activity 4</b> on the Fibonacci Number Sequence.	<ul style="list-style-type: none"> <li>• 1</li> <li>• 2</li> <li>• 3</li> <li>• 5</li> <li>• 8</li> <li>• 13</li> </ul>	<b>Note:</b> The Fibonacci sequence is {0, 1, 1, 2, 3, 5, 8, 13, , , }, hence the explanation of no rabbits in the first month.	
» Can you see a pattern developing?	<ul style="list-style-type: none"> <li>• If we add together any two consecutive terms we get the next term.</li> </ul>	<b>Note:</b> To explain why the first term is zero: explain that there were no rabbits in the first month. Then, there was 1 pair in the second month etc. A more complicated explanation of the Fibonacci rabbit problem can be found at <a href="http://en.wikipedia.org/wiki/Fibonacci_number">http://en.wikipedia.org/wiki/Fibonacci_number</a>	» Did the students recognise the pattern?
» Now, I want you all to try the interactive quiz called "Arithmetic Sequence Quiz" on the Student's CD.		» If computers are not available, hard copies of the quiz can be printed for the students.	
<p><b>Reflection:</b></p> <p>» Describe an arithmetic sequence.</p> <p>» Write down two formula used in connection with arithmetic sequences and what does each letter used mean?</p> <p>» Write down any questions you have on this section of the course?</p>	<ul style="list-style-type: none"> <li>• The difference between any term and the preceding term is always the same or <math>T_n - T_{n-1}</math> is a constant for all the terms.</li> <li>• <math>T_n = a + (n-1)d</math>  <math>T_n - T_{n-1} = d</math> (The common difference)  <math>a</math> = first term  <math>d</math> = common difference  <math>n</math> = the term  <math>T_n</math> = the <math>n^{\text{th}}</math> term  <math>T_{n-1}</math> = the term before the <math>n^{\text{th}}</math> term.</li> </ul>	» Circulate and take note of any questions or difficulties students have noted and help them answer them.	

# Section A: Student Activity 1

1. A gardener buys a plant that is 12cm in height. Each week after that the plant grows 10cm. Note: The plant is 12cm high at the beginning of the first week.
  - a. What will be the height of the plant at the beginning of the 1<sup>st</sup>, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> weeks, if it follows the same pattern?
  - b. Use graph paper to represent the pattern from part a.
  - c. Explain your reasoning.
  - d. What two pieces of information were you given initially?
  
2. The ground floor of a building is 6 metres in height and each floor above it is 4 metres in height.
 



  - a. Taking the ground floor as floor one. List the height of the building at each of the first five floors.
  - b. Use graph paper to represent the pattern from part a.
  - c. Explain your reasoning.
  - d. What two pieces of information were you initially given?
  
3. John has €500, and wants to go on a holiday, but does not have sufficient money. He decides to save €40 per week for a number of weeks to make up the deficit.
  - a. Given that he has €500 at the beginning of the first week, how much will he have saved at the end of the, 2<sup>nd</sup>, 3<sup>rd</sup>, 4<sup>th</sup> and 5<sup>th</sup> weeks?
  - b. Use graph paper to represent the pattern from part a.
  - c. Explain your reasoning.
  - d. What two pieces of information were you given initially?
  
4. What have the previous three questions got in common?

# Section A: Student Activity 1 (continued)

5. Find  $T_{10}$  and  $T_{12}$  for questions 1, 2 and 3 above.

	Formula for $T_{10}$	Value of $T_{12}$	Formula for $T_{12}$	Value of $T_{12}$
Question 1				
Question 2				
Question 3				

6. Find  $T_1, T_2, T_3, T_5, T_{10}$ , and  $T_{100}$  of an arithmetic sequence where  $a = 4$  and  $d = 6$ .

$T_1$	$T_2$	$T_3$	$T_5$	$T_{10}$	$T_{100}$

7. Find  $T_1, T_2, T_3, T_5, T_{10}$ , and  $T_{100}$  of an arithmetic sequence where  $a = 3$  and  $d = -2$ .

$T_1$	$T_2$	$T_3$	$T_5$	$T_{10}$	$T_{100}$

8. Find  $T_1, T_2, T_3, T_5, T_{10}$ , and  $T_{100}$  of an arithmetic progression where  $a = 3$  and  $d = \frac{1}{2}$ .

$T_1$	$T_2$	$T_3$	$T_5$	$T_{10}$	$T_{100}$

9. Find the 7<sup>th</sup>, 8<sup>th</sup> and 14<sup>th</sup> term in the arithmetic sequence 2, 6, 10, 14, ...

7 <sup>th</sup> Term	8 <sup>th</sup> Term	14 <sup>th</sup> Term



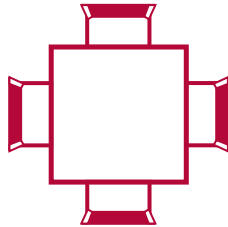
# Section A: Student Activity 1 (continued)

10. A baker has 400kg of flour on the first day of the month and uses no flour that day. How much will he have at closing time on the 17<sup>th</sup> day of the month, if he uses 24kg each day? Show your calculations using the formula for  $T_n$  of an arithmetic sequence.
11. Marjorie is trying to increase her fitness through exercise. The first day she walks 1500 metres and every day after that she increases this distance by 110 metres. How far will she be walking on the 12<sup>th</sup> day? Show your calculations using the formula for  $T_n$  of an arithmetic sequence.
12. Find the first 5 terms in each of the following sequences and determine which of the sequences are arithmetic assuming they follow the same pattern.

Sequence	$T_1$	$T_2$	$T_3$	$T_4$	$T_5$	Is the sequence an arithmetic sequence? Explain your answer.
$T_n = n + 4$						
$T_n = 2n$						
$T_n = 6 + n$						
$T_n = 9 + 2n$						
$T_n = 2n - 6$						
$T_n = n^2$						

13. On 1st November Joan has 200 sweets in a box. She eats no sweets the first day and she eats 8 sweets per day from the box thereafter. On what day of the month will she have 40 sweets left? Show your calculations using the formula for  $T_n$  of an arithmetic sequence.
14. Joan joins a video club. It costs €12 to join the club and any video she rents will cost €2. Jonathan joins a different video club where there is no initial charge, but it costs €4 to rent a video. Represent these two situations graphically and in algebraic form. Do either or both plans follow the pattern of an arithmetic sequence? Explain your reasoning.

## Section B: Student Activity 2

1. An art collection is valued at €50,000 in 2001 and its value increases by €4,000 annually.
  - a. Show that, as time passes, the value of the collection follows an arithmetic sequence? List the first ten terms of the sequence and give the first term and common difference.
  - b. How much will the collection be worth in 2051?
  
2. A new car was valued at €21,000 on 1<sup>st</sup> January 2001, and depreciated in value by a regular amount each year. On 1<sup>st</sup> January 2008, it had a value of zero. Show that the value of the car could follow an arithmetic sequence. How many terms are there in this sequence? By how much did it depreciate each year? Show your calculations.
  
3. The sum of the interior angles of a triangle is  $180^\circ$ , of a quadrilateral is  $360^\circ$  and of a pentagon is  $540^\circ$ . Assuming this pattern continues, find the sum of the interior angles of a dodecagon (12 sides). Show your calculations. <http://www.regentsprep.org/Regents/math/algtrig/ATP2/SequenceWordpractice.htm>
  
4. A restaurant uses square tables and if one person sits on each side how many people can sit at each table?
 
  - a. If two tables are pushed together, how many people can be seated? Draw a diagram.
  - b. If three tables are placed together in a straight line, how many people can be seated? Draw a diagram.
  - c. If 4 tables are placed together in a straight line how many people can be seated?
  - d. Is the pattern in this question an arithmetic sequence? Explain your answer.
  
5. The seats in a theatre are arranged so that each row accommodates four people less than the previous row. If the first row seats 50 people, how many will the 9<sup>th</sup> row seat?
  
6. The seats in a theatre are arranged so that each row accommodates four people more than the previous row. If the first row seats 50 people, how many will the 9<sup>th</sup> row seat?

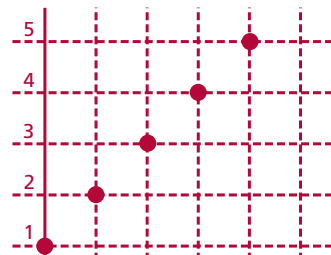
## Section B: Student Activity 2 (continued)

7. Find  $T_n$  of the following arithmetic sequence 12, 8, 4, ... For what value of  $n$  is  $T_n = -96$ ?

8. a. The pattern represented in the above graph is of an arithmetic sequence. Explain why this is so.

b. What is the 8<sup>th</sup> term of this sequence?

c. Write a story that this pattern could represent.



9. If the Mean Synodic Period of new moons (Period from new moon to new moon) is approximately 29.53 days. If the first new moon occurred on the 11<sup>th</sup> day of 2009, predict the day of the year on which the 5<sup>th</sup> new moon occurred in 2009. Show your calculations using appropriate formula.

10. How many three-digit positive integers exist so that when divided by 7, they leave a remainder of 5 if the smallest such number is 110 and the largest is 999? Show your calculations using appropriate formula.

11. If  $2k$ ,  $5k-1$  and  $6k+2$  are the first 3 terms of an arithmetic sequence, find  $k$  and the 8<sup>th</sup> term.

12. The sum of three consecutive terms in an arithmetic sequence is 21 and the product of the two extreme numbers is 45. Find the numbers.

13. Prove that the sequence  $T_n = 2n + 6$  is an arithmetic sequence.

14. Prove that the sequence  $T_n = n^2 + 3$  is not an arithmetic sequence.

15. If Marlene saved €40 per week for the first 8 weeks and then saved €50 per week for the next 8 weeks. Represent this graphically and explain why her savings pattern for the whole 16 weeks does not form an arithmetic sequence.

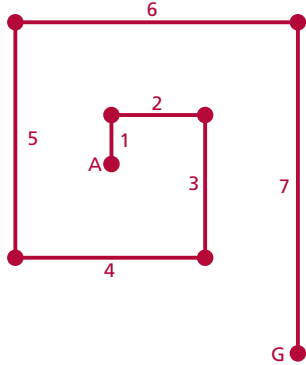
## Section C: Student Activity 3

*(Calculations must be shown in all cases.)*

- 1. The tenth term of an arithmetic sequence is 40 and the twentieth term is 30. Find the common difference and the first term.**
- 2. A doctor asked a patient to reduce his medication by a regular number of tablets per week. The reduction plan begins one week after the visit to the doctor. If 10 weeks after the visit to the doctor the person is taking 40 tablets and 20 weeks after the visit the person is taking 30. Find the weekly reduction in his consumption of tablets.**
- 3. Patricia is climbing up a large number of steps and to encourage her on her way, her friend started counting the number of steps she completes every minute. From then on she keeps a regular pace. After 2 minutes she has climbed 56 steps in total and after 8 minutes she has climbed 158 steps in total.**
  - a. Find the number of steps she climbs every minute once her friend started counting.**
  - b. Find the number of steps she had climbed before her friend started counting.**

If she had not maintained a regular pace, would you still have been able to use this method?
- 4. If the 9<sup>th</sup> floor of a building is 40 metres above the ground and the ground floor is 4 metres in height and each floor apart from the ground floor has equal height. Find the height of each floor. (Note the 1<sup>st</sup> floor is the one above the ground floor etc.)**
- 5. On the first day of a month a baker receives a delivery of flour and starts using this flour at the beginning of the following day. He uses a regular amount of flour each day thereafter. At the end of the 4<sup>th</sup> day of the month he has 1,000kg of flour remaining and at the end of the 12<sup>th</sup> of the month 640kg remain. How much flour does he use per day and how much flour did he have delivered? Use equations to represent this information and then solve the equations.**
- 6. A woman has a starting salary of €20,000 and after the first year she gets an annual increase of €2,000 per year. Find her annual salary when she reaches retirement age, 40 years from when she started the job?**

## Section C: Student Activity 3 (continued)

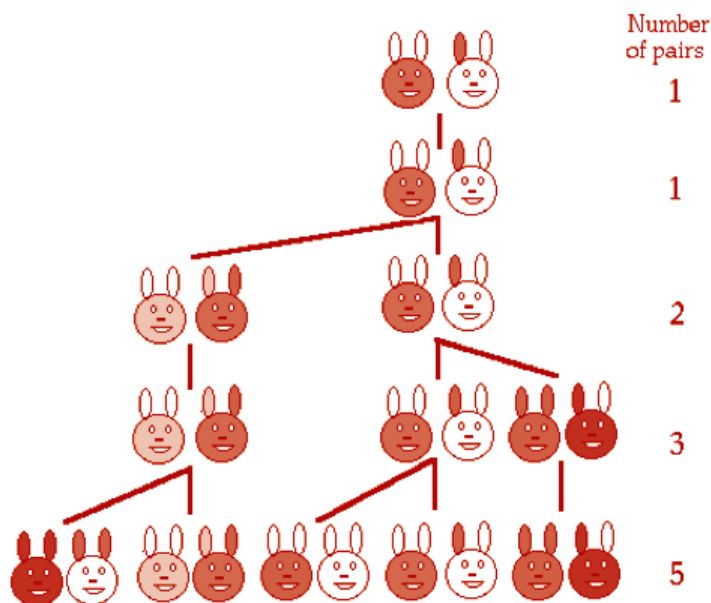
7. One week after joining a gym a man starts losing a constant amount of weight. Eight weeks after joining the gym he weighs 98kg and 15 weeks after joining the gym he weighs 91kg, find his weight before joining the gym and the amount of weight he lost each week. Is a weight loss programme a good application for an arithmetic sequence? Explain your answer.
8. a. Why is the pattern in the following drawing an arithmetic sequence and what will the length of the 88<sup>th</sup> side be?  
b. List an arithmetic sequence of your choice and draw a geometric representation of it.
- 
9. A new company made a profit of €2,000 in the 8<sup>th</sup> week of business and in the 12<sup>th</sup> week of business they made €3,000. If their accountant informed you that the increases in their profit followed an arithmetic sequence pattern, how much profit did they make in the first 4 weeks of business?
10. The membership fee for a gym is €100 and for each visit after that the charge is €10.
- a. How much will ten visits cost? Show your calculations.  
b. What will be the cost of joining the gym and visiting it on  $n$  occasions?
11. A walker is already walking 5km per day and decides to increase this amount by 0.1km per day starting on 1<sup>st</sup> August.
- a. What distance will he be walking on 3<sup>rd</sup> August?  
b. Is this an arithmetic sequence? Explain your reasoning.  
c. What is the formula for the  $n^{\text{th}}$  term?  
d. If this question had stated that the walker walked 5km on 1<sup>st</sup> August and that he had increased this by 0.1km, each day thereafter. What would the formula have been for:  
i. The 16<sup>th</sup> August?  
ii. The  $n^{\text{th}}$  day of the month
12. On 1<sup>st</sup> June, Joseph has 200 sweets in a box. He eats 8 sweets per day from the box starting on the 1<sup>st</sup> June. On what day of the month will he have 40 sweets left? Show your calculations using algebra. How does this differ from question 13 in **Section A: Student Activity 1**?

## Section D: Student Activity 4

Fibonacci was an Italian mathematician, who worked on this problem in the 13th century. He studied how fast rabbits breed in ideal circumstances. He imagined a newly born pair of rabbits, one male and one female were placed in a field and made the following assumptions about the mating habits of rabbits:



- Rabbits mate at exactly one month old and mate every month after that.
- Rabbits always have litters of exactly one male and one female.
- The gestation period for rabbits is exactly one month. (Not true)
- Rabbits never die. (Not true)



©<http://www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/fibnat.html#Rabbits>

- 1 How many pairs will there be after 1 month? Explain.
- 2 How many pairs will there be after 2 months? Explain.
- 3 How many pairs will there be after 3 months? Explain.
- 4 How many pairs will there be after 4 months? Explain.
- 5 How many pairs will there be after 5 months? Explain.
- 6 How many pairs will there be after 6 months? Explain.
- 7 Can you see a pattern developing? List the first 20 numbers in the Fibonacci Number Sequence?