Teaching & Learning Plans

Plan 3: Fair Trials with Two Dice

Leaving Certificate Syllabus

Project Maths Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. Student Learning Tasks – Teacher Input: This section focuses on teacher input and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. Student Activities – Possible and Expected Responses: Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. Teacher's Support and Actions: Gives details of teacher actions designed to support and scaffold student learning.

iv. Checking Understanding: Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan 3: Fair Trials with Two Dice

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Aims

- To learn that not all results are equally probable
- To consider a given set of rules and the outcomes that produce a 'win' for one of two players and to be able to determine whether the game is 'fair', i.e. whether each of the two players has the same chance of winning
- To consider the theoretical probability of each result and devise a set of rules to make the game 'fair'

Prior Knowledge

Students should have prior knowledge (from T and L Plan 1 and/or from primary school) of some terms associated with chance and uncertainty. They should be familiar with probability expressed as fraction or decimal in the range 0 to 1, or as a percentage in the range 0% to 100%.

Learning Outcomes

As a result of studying this topic, students will be able to

- investigate further what the concept of fairness means in a game with 2 dice
- list all the possible outcomes for throwing 2 dice using a two way table
- relate the number of outcomes to the fundamental principle of counting
- come up with rules for a game which make it fair/unfair
- construct a probability table
- understand the relationship between an event and its complement
- determine the probability of an event using the results of an experiment and use this to predict the result of a repetition of the experiment, for equally likely outcomes

Relationship to Leaving Certificate Syllabus

Sub-topics	Foundation Level	Ordinary Level	Higher Level
1.1 Counting	List outcomes of an experiment. Apply the fundamental principle of counting.	Count the arrangements of distinct objects.	
1.2 Concepts of probability	Recognise that probability is a measure on a scale of 01- of how likely an event is to occur.	Recognise the role of expected value in decision making with a focus on fair games.	Extend the students' understanding of the basic rules of probability.
Discrete probability (as relative frequency)	Probability of desired outcomes in problems involving experiments, such as, dice throwing.		
1.3 Outcomes of random processes	Apply the principle that, in the case of equally likely outcomes, the probability is given by the number of outcomes of interest divided by the total number of outcomes.		

Resources Required

Counters, 2 dice and a Game Sheet for each pair of students

Lesson Interaction			
Student Learning Tasks: Teacher Input	**Student Activities: Possible and Expected Responses**	**Teacher's Support and Actions**	**Checking Understanding**
» In each group of two, one person is nominated as A and one as B. Players A and B take turns to roll the die, and the winner is determined by the sum of the numbers on the faces as follows: » Students A and B alternately roll the die, each time adding the scores on each die to get the outcome. They place a counter on each outcome. » A wins if sum (i.e. outcome) is 2, 3, 4, 10, 11 or 12. » B wins if sum i.e. is 5, 6, 7, 8, 9. Play the game on **Game Sheet 1**.		» Distribute **Student Activity 1** so that students can write their prediction (**Student Activity 1A**) and keep a count of the number of times each player wins (**Student Activity 1B**). » Distribute counters and **Game Sheet 1** and dice to each pair, checking that students have a written prediction and a reason for it.	» Can students record their initial results in their copies?
» Think about this for a few minutes and write down which player you think will win the game most often if it is played many times, and why you think this.	» Students may think that since 11 numbers have been listed, and since A can win on 6 of these and B only on 5, that A is more likely to win than B.	» Ask for a show of hands on who thinks A will win and who thinks B will win.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» The game is played until one person has reached the bottom of the grid on **Game Sheet 1** and students count the number of times A won and the number of times B won and fill out the **Master Sheet** on the board also.	» The students should find that A wins only about 1/3 of the time.	» Check that each pair understands what they are doing and occasionally, as you circulate, ask "hands up how many A's are winning?" and then "hands up how many B's are winning?" so students are aware of an overall trend. » Place **Master Sheet 1** (results from the whole class) on the board.	» Were students able to conduct the game successfully and did they notice B winning much more often? » How many students in the class were able to make accurate predictions with valid reasons? » Did they react to this as being unfair?
» When you are finished fill out **Student Activity 1B** and fill in your results on **Master Sheet 1** (results from the whole class) on the board.	» Students check this against their prediction, analysing any difference and fill in **Student Activity 1C**. » The relative frequency is now closer to 1/3.	» As you circulate check predictions against outcomes of the experiment, and ask students about their justifications.	
» Which number on each die cannot be a possible outcome?	• "1 "can never be an outcome as the smallest sum possible is 2.		
» Does this game appear to be fair?	• No. Fair – if A and B have an equal chance of winning otherwise unfair.	» Give students a minute to think and then ask one student.	

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Why is it not fair?	• Students might say that 2 can only be got 1 way (1+1) whereas 9 for example can be got from (3+6), (4+5). • All outcomes are not equally likely.	» Give students a minute to think and then ask one student.	» Do students understand the concept of fairness?
» Is the outcome 4+5 the same as the outcome 5+4?	• No, because the first time 4 is from die 1 and the second time it's from die 2.	» Give students a minute to think and then ask one student.	
» In how many ways could an outcome of 9 be achieved?	• 4+5 • 5+4 • 3+6 • 6+3	» Give students a minute to think and then ask one student.	
» Could you design a table to show you all the possible outcomes. » The set of all the possible outcomes is called the 'Sample Space'. » Try this for a few minutes.	» Some students will probably come up with the 2 way table and show it to the class.	» Circulate looking at student suggestions and giving helpful hints. On finding a suitable table ask students to show and explain it to the class. Students may need help designing the table.	» How many of the students were able to come up with a suitable table?
» Individually, fill in the 2-way table for the sample space for the sum achieved on throwing 2 dice (**Student Activity 2A**).	» Students fill up the table as per example in **Student Activity 2A**.	» Distribute **Student Activity 2**. » Circulate to check that students understand how to fill it in.	» Are all students able to fill in the table successfully and with understanding?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» How many possible outcomes are there? **Student Activity 2B.**	• 36.	» Circulate and look at the answers being filled in, asking questions where necessary.	
» Can you relate this back to the fundamental principle of counting in a previous lesson?	• If there are 6 different outcomes from die 1, and 6 different outcomes from die 2, then the total number of possible outcomes is 6x6 = 36.	» Ask the question and wait for someone to volunteer an answer. If it's the same person answering all the time, ask another student, leading them to the answer if they don't know it, by referring back to an example they did when learning the fundamental principle of counting.	» Have students been able to recall and use the fundamental principle of counting?
» Going back to the rules of the game, for how many outcomes will player A win and for how many outcomes will player B win? **Student Activity 2C.**	• 12 of the 36 outcomes give a win for A whereas 24 of the outcomes give a win for B.		
» Go back to your prediction again, **Student Activity 1C**, and if it was not consistent with the outcome write down why it was different.	» Students articulate their misconceptions.	» Circulate and note misconceptions, and ask a selection of students who thought A would win to explain why they thought that and why they now think differently.	» Are students able to understand in cases where their predictions were in error why this was so?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» Write down new rules for the game, **Student Activity 2D**, which will make it fair and an explanation of why it is now fair. » Is more than one set of rules possible?	» Different groups present their new rules for class approval. • For example. Player A wins if the outcome is 2, 3, 4, 5, 8 or 10 and B wins if the outcome is 6, 7, 9, 11 or 12.	» Circulate and when groups are finished ask a couple of groups to explain their new rules.	» Are students able to make up more than one set of rules satisfying the criterion for a fair game?
» The approach in Teaching & Learning Plan 2, and above in **Student Activity 1B**, is known as the 'experimental' or 'empirical' approach to calculating probabilities. In this case the *probability* of an event is the value that the relative frequency tends to in an infinite number of trials. » In the theoretical approach, when throwing a die, there are six possible outcomes which are all *equally likely* to appear due to the symmetry of the die, and given that it is a fair die (not loaded) so we have no reason to assume that any number will appear more often than another. A2 is one of the 6 possible outcomes, so the likelihood of it turning up is 1 out of 6.			

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» What is the relationship between the experimental approach to calculating probability and the theoretical approach?	• As the number of trials increases the relative frequency becomes almost equal to the theoretical probability.	» Asks students to recall and then after a short pause asks an individual.	» Can students recall how to calculate probability for equally likely outcomes?
» How do we calculate the theoretical probability of each outcome in this sample space?	• How many times it occurs divided by the total number of possible outcomes.		
» What assumption are we making?	• All outcomes are equally likely.		
» Are all 36 outcomes equally likely here?	• Yes.		
» Construct a probability table (**Student Activity 3**) for the sum of 2 dice using **Student Activity 2A**.		» Walk around observing students as they fill out the probability table.	» Did students fill out the table correctly?
» Assuming that both of the dice are fair, and all 36 outcomes are equally likely, what is the probability of the sum being 5?	• Using the 2 way table (**Student Activity 2A**) showing the sample space constructed for the game, there are 4 outcomes for 5, so the probability of getting a 5 is 4/36.		
» What is the sum of the probabilities for the sample space?	• 1	» Ask students if they remember this happening in the last lesson with outcomes from 1 die?	» Did all students get 1?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» If the probability of getting a 3 is 2/36, what is the probability of not getting a 3 without adding up all the other probabilities? Write down the answer.	• 1-2/36=34/36	» If a student cannot get the answer without adding up all the other probabilities, tell them to do that first and then see if they can then find the other way by inspecting their answer.	» How many students got the correct answer without having to add up all the other probabilities?
» Going back to the original rules for the game, what is the probability that A wins? » Fill in answer on **Student Activity 3**.	• 12/36=1/3	» For students who have difficulty: ask them to count how many of the outcomes give A a win, and how many outcomes are there in total.	
» Use the probability that A wins to calculate the probability that B wins, without adding. » Fill in answer on **Student Activity 3**.	• 1-1/3=2/3	» If a student cannot get the answer without adding up all the other probabilities, tell them to do that first and then see if they can then find the other way by inspecting their answer.	» Do students understand the relationship between an event and its complement?
» Compare this with experimental result for relative frequency. » Fill in answer on **Student Activity 3**.	• They are very close and the relative frequency becomes closer to the theoretical probability as the number of games increases.	» Circulate checking that **Student Activity 3** is being completed, asking questions of students who are having difficulty.	» Do students see that the experimental results approach the theoretical values for probability as the number of games increases?

Student Learning Tasks: Teacher Input	Student Activities: Possible and Expected Responses	Teacher's Support and Actions	Checking Understanding
» If the probability of a sum of 7 occurring is 6/36=1/6, how many 7's would you expect to get if the dice are tossed 100 times?	• 100x1/6 i.e. approx 17 (1/6 of the time you expect a 7.)	» Emphasise that you are dealing with equally likely outcomes.	» Can students calculate an expected value for a large number of trials?
Reflection » Write down 3 things you learned about probability today. » Write down anything you found difficult. » Write down any questions you may have.	• How to: 1. list all possible outcomes (sample space) for throwing 2 dice 2. use the list of all possible outcomes to judge fairness 3. construct a probability table 4. calculate the complement of an event 5. calculate expected value for a large number of trials.	» Circulate and take note particularly of any questions students have and help them to answer them.	» Have all students learned and understood these items? » Are they using the terminology with understanding and communicating with each other using these terms?

Game Sheet 1

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	A wins	A wins	A wins	B wins	B wins	B wins	B wins	B wins	A wins	A wins	A wins
1	2	3	4	5	6	7	8	9	10	11	12

Student Activity 1

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Student Activity 1A
Prediction

Player _____ will win most often because:

Student Activity 1B

Play the game and record the results below:

	Use Tally Marks to Help You Keep a Score	Total (Frequency)	Relative Frequency (total no. of wins/total no. of games)
Player A Wins			
Player B Wins			
Totals			

Student Activity 1C

Did your predicted results agree with your actual results?

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Master Sheet 1 (Results from the Whole Class)

Play the game and record the results below:

	Use Tally Marks to Help You Keep a Score	Total (Frequency)	Relative Frequency (total no. of wins/total no. of games)
Player A Wins	2+3+...		
Player B Wins			
Totals			

Student Activity 2

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Student Activity 2A

Complete the table to show all of the possible outcomes.
Two way table showing the sample space.

Number thrown	1	2	3	4	5	6
1	1+1=2					
2						
3						
4						
5		5+2=7				
6						

Student Activity 2B

How many possible outcomes are there?_____

Student Activity 2C

Original Rules: **Player A wins when the sum is 2, 3, 4, 10, 11 or 12.**

Player B wins when the sum is 5, 6, 7, 8 or 9.

For how many outcomes will player A win?_____

For how many outcomes will player B win?_____

Student Activity 2D

New Rules: **Player A wins when the sum is** _____

Player B wins when the sum is _____

Why I chose these new rules:

_____

_____

_____

Student Activity 3

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Sum	Frequency	Probability $= \frac{no.\ of\ outcomes\ in\ the\ event}{no.\ of\ outcomes\ in\ the\ sample\ event}$
2	1	$\frac{1}{36}$
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		

