Teaching & Learning Plans

Ratio and Proportion

Junior Certificate Syllabus

Project Maths
Tionscadal Mata
Development Team

The Teaching & Learning Plans are structured as follows:

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Aims outline what the lesson, or series of lessons, hopes to achieve.

Prior Knowledge points to relevant knowledge students may already have and also to knowledge which may be necessary in order to support them in accessing this new topic.

Learning Outcomes outline what a student will be able to do, know and understand having completed the topic.

Relationship to Syllabus refers to the relevant section of either the Junior and/or Leaving Certificate Syllabus.

Resources Required lists the resources which will be needed in the teaching and learning of a particular topic.

Introducing the topic (in some plans only) outlines an approach to introducing the topic.

Lesson Interaction is set out under four sub-headings:

i. **Student Learning Tasks – Teacher Input:** This section focuses on possible lines of inquiry and gives details of the key student tasks and teacher questions which move the lesson forward.

ii. **Student Activities – Possible Responses:** Gives details of possible student reactions and responses and possible misconceptions students may have.

iii. **Teacher's Support and Actions:** Gives details of teacher actions designed to support and scaffold student learning.

iv. **Assessing the Learning:** Suggests questions a teacher might ask to evaluate whether the goals/learning outcomes are being/have been achieved. This evaluation will inform and direct the teaching and learning activities of the next class(es).

Student Activities linked to the lesson(s) are provided at the end of each plan.

Teaching & Learning Plan:

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Aim

- To enable students understand the concepts of ratio and proportion

Prior Knowledge

Students have prior knowledge of natural numbers, integers, fractions, decimals and percentages as well as factors, multiples and primes and will have met simple ratios and rate problems in primary school. They will also have met probability in primary school which involves ratio.

Learning Outcomes

As a result of studying this topic, students will be able to:

- distinguish between absolute comparison and relative comparison
- see ratios as comparing part to part and fractions as comparing part to whole, where the quantities being compared have the same units.
- see rates as the ratio of two quantities having different units
- appreciate the importance of order when dealing with ratios
- find equivalent ratios
- divide a number into a given ratio
- recognise a proportion as a statement of equivalent ratios 5:2 = 10:4 or set up a proportion to find x as in 5:2 = 8:x
- distinguish between proportional and non-proportional situations recognising the multiplicative relationship that exists between the quantities in proportional situations, examinable at a later stage using tables, graphs and algebraic expressions
- use a variety of techniques including the unitary method, factor of scale and tables, to solve proportional tasks and to recognise that these techniques are all related
- solve problems involving proportional reasoning in different contexts.

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Relationship to Junior Certificate Syllabus

Topic Number	Description of topic *Students learn about*	Learning outcomes *Students should be able to*
3.1 Number Systems	Students learn strategies for computation that can be applied to any number. Implicit in such computational methods are generalisations about numerical relationships with the operations being used. Students articulate the generalisation that underlies their strategy, firstly in the vernacular and then in symbolic language. Problems set in context, using diagrams to solve the problems so they can appreciate how the mathematical concepts.	• consolidate their understanding of the relationship between ratio and proportion.

Relationship to the Leaving Certificate Syllabus

Strand 3: Number

Strand 3 further develops the proficiency learners have gained through their study of Strand 3 at junior cycle. Learners continue to make meaning of the operations of addition, subtraction, multiplication and division of whole and rational numbers and extend this sense-making to complex numbers. They extend their work on proof and become more proficient at using algebraic notation and the laws of arithmetic and induction to show that something is always true. They utilise a number of tools: a sophisticated understanding of proportionality, rules of logarithms, rules of indices and 2-D representations of 3-D solids to solve single and multi-step problems in numerous contexts.

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Resources Required

1cm squared paper.

Real Life Context

The following examples could be used to explore real-life contexts:

- ratio of the length of the diagonal to side length in a square
- ratio of the length of the circumference to length of the diameter in a circle
- Fibonacci numbers and the nautilus shell
- rate problems such as in speed, pricing, currency conversions etc.
- slopes (rates of change)
- photographic enlargements
- scaling in maps.

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Teacher Reflections

Lesson Interaction

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section A: The difference between relative and absolute comparisons and the concept of ratios and proportions			
» Work in pairs on these two questions. Discuss whether or not they are the same kind of problem. » If 4 copies cost €2, how much would 8 copies cost? » When Bríd was 4 years old her younger brother was 2 years old. Bríd is now 8 years old. How old is her younger brother?	• As you have twice as many copies they will cost twice as much. 8 copies cost €4. However, if we use the same reasoning with Bríd's brother the gap between their ages would then be 4 years when it should always be 2 years. Bríd is 4 years older now so her brother is also 4 years older and is now 6. • This question is different as we worked out the difference in the ages and not how many "times" one was older than the other.	» Write the two problems on the board. » Where students have difficulty understanding the differences ask them to draw up tables of values for each situation.	» Can students distinguish between additive and multiplicative reasoning?
» We are using "relative" comparison for the copy prices and "absolute" comparison for the ages. » Can you think of other similar examples?	• Changing money to a different currency is an example of relative comparison. If 2 cars were travelling at the same speed but one started out ahead of the other, they remain the same distance apart. This is an example of absolute comparison.	» Give students some hints of proportional versus non proportional situations if they cannot initially think of their own.	» Can students come up with some examples?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Kiera is older than Michael. Due to the recession, Kiera's pocket money is cut from €60 to €50. Michael's money is cut from €20 to €10. Michael says this is unfair. Why would he say this if they both drop €10? Discuss this in pairs.	• Kiera originally got 3 times the amount Michael got, (60 compared to 20), but now she gets 5 times the amount he gets, (50 compared to 10). • Kiera's pocket money was cut by $\frac{1}{6}$ whereas Michael's was cut by $\frac{1}{2}$. • They were not cut in proportion even though they were reduced by the same amount.	» Circulate among students and where students have difficulty ask them if, for example, it would be fair to take €100 tax per week from a person who earns €200 per week and the same tax of €100 per week from a person who earns €500 per week? » Discuss with students how relative comparison seems to be fairer than absolute comparison in this case.	» Are students again distinguishing between additive and multiplicative reasoning? » Can students understand from a real life example what it means to reduce proportionally? » Are some students using the phrase "in proportion"?

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» When we compare €60 to €20, using the idea of how many "times" €60 is "more" than €20 instead of the difference between €60 and €20, we call this the ratio 60:20.		» Write the word "ratio" on the board and 60:20	» Do students see the link between creating equivalent fractions and equivalent ratios?
» How does 60 compare to 20?	• 60 is 3 times 20.	» Remind students of how they created equivalent fractions not by subtracting or adding the same number to the numerator and denominator but by multiplying or dividing the numerator or denominator by the same number.	
» Write 60:20 as a ratio.	• 60:20 = 3:1		
» Write down other numbers in the same ratio as 60:20.	• 60:20 = 30:10= 15:5 = 12:4 = 6:2= 3:1 = 120:40 etc.		» Can students simplify ratios?
» What is the simplest form of the ratio 60:20? Why?	• 3:1 is simplest because the only factor they have in common is 1.		» Can students see that there are an infinite number of equivalent ratios for any given ratio?
» How did you generate ratios equivalent to 60:20?	• By thinking of any pair of numbers where one was three times greater than the other. By multiplying or dividing each number in the ratio by the same number.		
» Where else did you multiply or divide a pair of numbers by the same number to create an equivalent form.	• When generating equivalent fractions.		
» What does that lead you to conclude about fractions?	• Fractions are ratios also.		
» When we have a statement of equivalent ratios, we have a **proportion**.			
» Set up a proportion for 6:4.	• 6:4 = 3:2 or 6:4 = 12:8 etc.		
» Set up the equivalent ratios for 60:20 as equivalent fractions.	• $\frac{60}{20} = \frac{30}{10} = \frac{15}{5} = \frac{12}{4} = \frac{6}{2} = \frac{3}{1}$		

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Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» What should Michael's pocket money (M) have been if it had been reduced in proportion to Kiera's (K)? Explain your answer. » What does 20:60 represent in this context?	• Keeping the 3: 1 (K:M)ratio 60:20 = 50:16.67 • This is the ratio of M:K	» Tell students to write down and clarify their thinking on this. Encourage them to think of real life contexts, for instance, the reasoning behind buying in bulk.	» Do students see that order is important in expressing ratios? 60:20 is not the same as 1:3.
» Comment on these ratios: 1. Pupil teacher ratio of 1:24 2. Doctors to patients in a hospital = 20:1 3. Players to fans at a game is 2000:1	• They are all the wrong way around. • There would be 24 teachers to each student, 20 doctors for every patient and 2000 players for each fan if they were correct.		» Are students appreciating the importance of order in ratios by making sense of real life examples? » Can students elaborate on the reality of the situation if these ratios were true?
» In ratios we compare two quantities with the same units by division. The resulting ratio has no units. » Can you suggest the first step to writing the following example as a ratio: 5 seconds: 5 minutes. » Simplify 5 km:5 miles if 1 km = 5/8 mile.	• Write both numbers in the same units first. 5 seconds: 300 seconds = 1:60 • 5 km:5 miles = 1km:1 mile = 5/8 m:1 m = 5:8	» Circulate as students are simplifying these ratios giving support where necessary.	» Do students understand that the numbers in a ratio must all be in the same units and that the ratio is then written without units?

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Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» 2/5 of the marks at the end of term are going for class work, 3/7 for homework and the remainder for the end of term test. What is the ratio of class work marks to homework marks using whole number ratios? What fraction of the total marks is for the test?	• 2/5:3/7 = 14/35:15/35 = 14:15 • 1- (29/35)=6/35 is for the test.	» Remind students of how they set up equivalent ratios by multiplying or dividing both numbers in the ratio by the same number. Circulate and give more help where needed.	» Are students able to simplify ratios given as fractions?
» Can you suggest ratios you could generate for the class? e.g. the number of students who travel by bus to school compared to those who walk. » Simplify the ratios where necessary.	• The number of boys to girls in the class. • The number of girls to the number of boys. • The number of students who like soaps to those who don't like soaps. • The number of students who support Manchester Utd. compared to those who support Liverpool.	» Give students some examples to get them started. » Encourage students to come up with the nearest equivalent simple ratio e.g. 13:7 is approximately 14:7 = 2:1. » Hand out **Section A: Student Activity 1**. » Have a class discussion on 1 or 2 of the examples, as indicated below, before getting students to work on the examples in small groups.	» Can students come up with varied suggestions for different ratios and ratios with more than 2 numbers?
» Look at the table of classes 2A and 2B in example 1. Which class has the greater proportion of girls? » Discuss with the person next to you for 2 minutes.	• There are a greater number of girls in 2B because 11 is more than 10. However, half of the class 2A is girls whereas less than half of the class 2B is girls. We therefore say that 2A has a greater proportion of girls than 2B.	» Hand out **SectionA: Student Activity 1**. » Put this table on the board.	» Are students distinguishing between the absolute and relative comparison?

Class	2A	2B
No. of students	20	26
No. of girls	10	11

<table>
<tr>
<th>Student Learning Tasks: Teacher Input</th>
<th>Student Activities: Possible Responses</th>
<th>Teacher's Support and Actions</th>
<th>Assessing the Learning</th>
</tr>
<tr>
<td>» Look at the table, from example 2, which is on the board and tell me which class you would choose if you were a Manchester Utd. fan and wanted to feel you were among like-minded people and why?

» Discuss with the person beside you for 2 minutes.</td>
<td>• Choose 1A because it has more United fans.

• Choose 1B because half of this class are Manchester Utd. fans whereas only 10 out of 30 which is ⅓ of Class 1A are Manchester Utd. fans and ½ > ⅓

• Only 1 out of every 3 students in 1A is a Manchester Utd. fan so I choose 1B where it is 1 out of every 2.

• Choose 1B as Manchester Utd. to Liverpool fans is 1:1 for 1B whereas it is 1:2 for 1A.</td>
<td>» Draw the table on the board
<table>
<tr><th></th><th>Man. Utd</th><th>Liverpool</th><th>Total</th></tr>
<tr><td>1A</td><td>10</td><td>20</td><td>30</td></tr>
<tr><td>1B</td><td>9</td><td>9</td><td>18</td></tr>
</table>

(See Table, Example 2 in Student Activity A).</td>
<td>» Are students giving one of the last 3 answers? Can they reason in different ways to come to the same conclusion?

» Are students using relative as opposed to absolute comparisons?</td>
</tr>
<tr>
<td>» What is different about comparing Manchester Utd. fans to Liverpool fans and Manchester Utd. fans or Liverpool fans to the whole class?

» Could we write both as ratios?

» Can we say from the first ratio that ½ of the class are Manchester Utd. fans?</td>
<td>• In the first case it is comparing a part to a part and in the second case it is comparing a part to the whole.

• Yes – for 1A, part (M) to part (L) is 10:20 = 1:2 and for part (M) to whole it is 10:30 = 1:3

• No because1:2 is the ratio of Manchester Utd. fans to Liverpool fans only rather than their ratio to the whole class.</td>
<td></td>
<td>» Are students applying what they learned about fractions to clarify differences and similarities between fractions and ratios?</td>
</tr>
</table>

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Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Fractions are called rational numbers because they express the ratio of a pair of integers where the denominator cannot be zero. Why not?	• You can't divide by zero	» Hold a discussion as to why you can't divide by zero.	
» Are percentages ratios? Explain.	• Yes because they compare a part to a whole of 100 parts		
» Work in pairs to complete **Section A: Student Activity 1**.		» Circulate and, as students complete each question, ask for feedback from different groups either verbally or on the board and look for class agreement.	» Are students using relative comparisons for ratios and are they able to simplify ratios?

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Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section B: Dividing a number into a given ratio			
» Sylvia is sharing a box of chocolates with her brother Dan. She says "2 for you and 3 for me" as she divides them out in the ratio 2:3. She continues until all the chocolates have been divided up. When she has finished she says to Dan "OK, you got 2/3 of the sweets." because I divided them in the ratio 2:3. Why is Dan frowning? Is there a difference between 2:3 and ⅔? Discuss. » Complete **Section B: Student Activity 2.**	• The ratio 2:3 compares part to a part and so Dan has ⅔ of Sylvia's sweets rather than ⅔ of the total. In fact, the sweets are divided into 5 equal portions of which Dan receives two and Sylvia three. So Dan's number of sweets is only ⅖ of the total.	» Allow students time to work this out in pairs. » If necessary, give students manipulatives such as counters to model the situation. » Discuss with the students why the amount of sweets that Dan receives is ⅔ of Sylvia's share. » Get feedback from the class when they have finished and agree a strategy for dealing with this type of problem. » Distribute **Section B: Student Activity 2**. » Help students with difficulties through probing questioning.	» Can students differentiate between fractions (always part to whole) and ratios which can represent part to part or part to whole comparisons? » Are students able to complete all of **Section B: Student Activity 2**? Do some students complete the last question by trial and error? » Are students checking their answer to the last question to ensure it fits the given criteria?

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Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section C: Ratios of more than two quantities			
» If $a{:}b$ = 3:5 and $b{:}c$ = 4:7, find the ratio of $a{:}b{:}c$.	• b is the common link. • We need the Lowest Common Multiple (LCM) of 5 and 4 which is 20. • 12:20:35	» Write the question on the board. » See if students come up with needing b as the "common link" and linking it with the LCM of 5 and 4. As this is a HL activity, give time to students before giving hints about LCM. » Ask them to make a list of equivalent ratios for each ratio if they are having difficulties.	» Are students making the link with how they got common denominators with fractions by finding the LCM of the denominators?
» If $\frac{x}{y} = 3$ and $y = 3z$ find $x : y : z$	• $x{:}y$ = 3:1 and $y{:}z$ = 3:1 y is common when looking for $x{:}y{:}z$ – hence find the LCM of 1 and 3 $x{:}y$ = 9:3 $y{:}z$ = 3:1 $x{:}y{:}z$ = 9:3:1	» Refer students to the last example and remind them of equivalent ratios.	» Can students convert from equivalent fractions back to a proportion (equivalent ratios)? » Are students applying knowledge about equivalent ratios?

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» If $\frac{1}{x}:\frac{1}{y}:\frac{1}{z}=2:3:4$ find the values of $x{:}y{:}z$ » Complete **Section C: Student Activity 3** in pairs.	$x:y:z=\frac{1}{2}:\frac{1}{3}:\frac{1}{4}$ $x:y:z=6:4:3$ or $\frac{1/x}{1/y}=\frac{2}{3}=\frac{y}{x}\therefore x:y=3:2$ $\frac{1/y}{1/z}=\frac{3}{4}=\frac{z}{y}\therefore y:z=4:3$ $\therefore x:y:z=6:4:3$	» Write the question on the board. » Allow students to explain different approaches and get a class consensus. » Distribute **Student Activity 3**.	

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Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section D: (a) The relationship between the length of the diagonal of a square and its side length (b) The relationship between the diameter of a circle and the length of its circumference			
» Working in pairs, investigate if there is a relationship between the length of the diagonal of a square and the length of its sides. Make a prediction as to whether or not you think there will be a constant ratio or if it will be different for all squares.	• I don't know if there will be a common ratio. • I expect there will be a common ratio. • I think the ratio will be different for every square. • The ratio of the length of the diagonal to the length of the side is 1.4: 1, correct to one decimal place.	» Distribute **Section D: Student Activity 4**. » Ask students if they can prove this for the general case. » If some students finish early, ask them to investigate the ratio of the perimeter of an equilateral triangle to its height or the ratio of the length of the diagonal of a rectangle to its perimeter. An additional exercise might include finding geometrical figures where such ratios are constant.	» Are students checking the result of their investigation against their prediction?

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<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr><td colspan="4">Section E: The ratio of consecutive numbers in the Fibonacci sequence</td></tr>
<tr><td>» Look at the first few terms of this infinite sequence of numbers,
1, 2, 3, 5, 8, 13, 21, 34, …

» Can you tell how to work out the next number in the sequence? Work on this in pairs for a few minutes.</td><td>• The next number is arrived at by adding up the two previous numbers.
Or
Adding up the two previous numbers gives the next number.</td><td>» Some students may need guiding questions to help them see the pattern.</td><td></td></tr>
<tr><td>» This sequence of numbers is named after the 13th century Mathematician Fibonacci (Leonardo of Pisa). He was given the nickname Fibonacci meaning son of Bonaccio.

» Working in pairs, calculate the next 8 Fibonacci numbers and calculate the ratio of successive numbers (2nd:1st, 3rd:2nd etc.) in the sequence. See if any pattern emerges if the answers are corrected to 3 decimal places.

» The answers all approach a value known as the golden ratio denoted by the Greek letter Φ which has the exact value
$\frac{1+\sqrt{5}}{2}$
which is approximately 1.61803398….</td><td><table>
<tr><td>1/1=1</td><td>55/34=1.6176</td></tr>
<tr><td>2/1=2</td><td>89/55 = 1.6182</td></tr>
<tr><td>3/2=1.5</td><td>144/89=1.61797</td></tr>
<tr><td>5/3=1.667</td><td>233/144=1.6181</td></tr>
<tr><td>8/5=1.6</td><td>377/233=1.618</td></tr>
<tr><td>13/8=1.625</td><td>610/377=1.618</td></tr>
<tr><td>21/13=1.615</td><td>987/610=1.618</td></tr>
<tr><td>34/21=1.619</td><td>1597/987=1.618</td></tr>
</table>
• The ratios are almost constant as the ratios of bigger and bigger consecutive Fibonacci numbers are calculated.</td><td>» Ask students to investigate the golden ratio on the internet and to report back their findings.</td><td>» Are students seeing that ratio can be a useful tool for investigating patterns?</td></tr>
</table>

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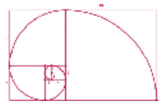
Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
 **Section F: The Fibonacci sequence, the golden rectangle and the link to the nautilus shell** The nautilus starts out as a very small animal which only needs a very small shell. The shell increases in spirals. Let's try and replicate a spiral increase on squared paper.			
» We can represent the growth of the spiralled shell as ¼ circles drawn in squares joined onto rectangles. We are going to investigate the ratios of the lengths of the sides of the rectangles generated and of the radii of the quarter circles. » Draw the spiral following the pattern on squared paper.	» Students follow the teacher as he/she generates more golden rectangles by adding squares to the longer side of the previously drawn golden rectangle and insert a quarter circle into each square.	» Distribute **Section F: Student Activity 5** » Show students how to draw initial parts of the shape below and see if they can continue themselves after the first few rectangles. 	» Given the initial few rectangles, are students able to continue to generate the pattern unaided?
» What are the lengths of the sides of the successive rectangles generated and the radii of the quarter circles drawn? » What do you know about these numbers? » All the rectangles formed in the construction of the spiral are golden rectangles as the ratio of their longer to their shorter sides is approximately 1.618. The golden rectangle is the only rectangle to which you can add on a square and get another similar rectangle (i.e. with sides in the same ratio). **Note** the meaning of the word "similar" in mathematics.	• 1, 1, 2, 3, 5, 8, 13, 21, 34, ... • They are Fibonacci numbers and the ratio of successive Fibonacci numbers approaches a number called the golden ratio.	» Show a step by step version of the drawing to students, using a dynamic geometry software package, after they have constructed it themselves. » A poster of one or more of the student drawings should be exhibited on the class room wall. » Ask students to make a note in their copies that "similar" means "in the same ratio".	» Can students distinguish between the use of the word 'similar' in mathematics meaning "in the same ratio" as opposed to a less defined use of the word in English?

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr><td colspan="4">Section G: Similar rectangles</td></tr>
<tr>
<td>» Look at this sheet of rectangles. What defines a rectangle?
» Do any of the rectangles look alike or similar?
» How could you check this?</td>
<td>• It is a parallelogram where all the angles are 90°.
• If you take measurements you may see that the sides of some are in the same ratio</td>
<td>» Distribute Section G: Student Activity 6 on similar rectangles.
» Depending on the class, the table could be left out to give less scaffolding to more able students.</td>
<td>» Where no table has been given, have students come up with the idea of finding the ratio of the sides of the rectangles to check for similarity?</td>
</tr>
<tr>
<td>» Measure the ratio of the shorter side to the longer side for each rectangle.

» Of all the figures A to K is there an odd one out?
» Is every square a rectangle? Is every rectangle a square?</td>
<td><table>
<tr><th></th><th>S</th><th>L</th><th>S:L</th><th>S:L</th></tr>
<tr><td>A</td><td>15</td><td>24</td><td>15:24</td><td>5:8</td></tr>
<tr><td>B</td><td>1</td><td>4</td><td>1:4</td><td>1:4</td></tr>
<tr><td>C</td><td>4</td><td>16</td><td>4:16</td><td>1:4</td></tr>
<tr><td>D</td><td>6</td><td>15</td><td>6:15</td><td>2:5</td></tr>
<tr><td>E</td><td>5</td><td>8</td><td>5:8</td><td>5:8</td></tr>
<tr><td>F</td><td>10</td><td>16</td><td>10:16</td><td>5:8</td></tr>
<tr><td>G</td><td>4</td><td>10</td><td>4:10</td><td>2:5</td></tr>
<tr><td>H</td><td>8</td><td>20</td><td>8:20</td><td>2:5</td></tr>
<tr><td>I</td><td>3</td><td>12</td><td>3:12</td><td>1:4</td></tr>
<tr><td>J</td><td>4</td><td>4</td><td>4:4</td><td>1:1</td></tr>
<tr><td>K</td><td>2.5</td><td>4</td><td>2.5:4</td><td>5:8</td></tr>
</table>
• Group 1: B,C,I –1:4 [J(1:1)]
• Group 2: D,G,H – 2:5
• Group 3: A,E,F,K – 5:8
• Odd one out – square 1:1

• The square is still a rectangle as it has all the properties of a rectangle but the other rectangles are not squares as they do not have 4 sides of equal length.</td>
<td>» Divide the class into groups giving each group several rectangles to measure e.g. group 1 measures rectangles B, C, I, J etc.
» Get students to fill in the table.
» Check that students are getting the ratios of corresponding sides.
» When the students have calculated the ratios, fill them in on the board in a table.
» Look for class agreement on measurements and get at least one set of measurements from each group.
» Mini white boards could be used to get measurements from all groups.
» It is useful to have an enlarged copy of the sheet of rectangles or show one using the data projector or overhead projector.</td>
<td>» Do any students suggest looking for equivalent ratios?</td>
</tr>
<tr>
<td>» Compare the areas of A and E</td>
<td>• 360 unit^2, 40 unit^2</td>
<td></td>
<td></td>
</tr>
</table>

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Teacher Reflections

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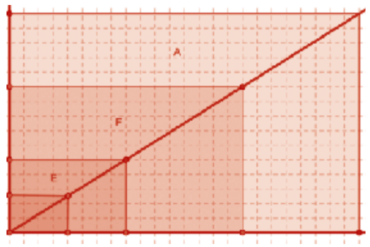
Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» What is the ratio of the side lengths of A to the corresponding side lengths of E? » What is the ratio of the area of A compared to the area of E? Explain the difference in these two ratios.	• Ratio of corresponding sides of A to E is 3:1 • Ratio of area of A to area of E = 9:1 • Area of A = 15 x 24 = (5 x 3) x (8 x 3) • Area of E = 5 x 8 • Area of A = 9 x (area of E)		» Do students see that when the side of a rectangle is enlarged by a factor of k that the area is enlarged by a factor of k^2?
» If you bought a 7″ pizza for €5 – what should you expect to pay for a 14″ pizza? » Explain your answer.	• Double • Less than double • More than double • Area for 7″ pizza = $\pi\,(7)^2$. • Area for 14″ pizza = $\pi\,(14)^2 = \pi\,(7)^2(2)^2 = 4(\pi(7)^2)$. • The area will go up by 4 – if paying in proportion you would pay €20.	» Remind students of the area of a circle formula and where to find it in the tables. » Ask students to find out typical prices. (Pizza sellers don't want to have arguments about proportional reasoning and would usually increase the price by a factor of somewhat less than 4 in this case.)	» Do students realise that buying in bulk can save you money as usually prices are not increased proportionally?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Cut out similar rectangles A, F, E, K. Stack them in the co-ordinated plane so that they are all aligned at one corner in increasing size with corresponding sides lined up - the common corners being placed at the orgin. » What do you notice about all the location of the opposite corners? » For every 8 units run (across) what is the rise of this line? » Express this as a ratio » This is the slope of the line. As the rectangles have equal ratio sides they are in proportion to one another.	 • They all lie on a line passing through (0,0) • For each 8 units run the rise is 5 units • Rise/run = 5/8 • Rise: run = 5:8= 5/8:1	» Show this on the board or use the data projector and a dynamic geometry software package. » For some classes, ask different groups to use rectangles with different proportions i.e. some groups use rectangles with sides in the ratio 1:4 or 2:5 or a ratio of their choice, to see if what they have seen with rectangles A, F, E, K applies in other cases.	» Are students associating equal ratios with "in proportion" as they did with the pocket money problem earlier?
» What are the coordinates of the points marked on the line and what do they represent?	• (4, 2.5), (8, 5), (16, 10), (24, 15). These pairs of numbers represent the width and height of similar rectangles and all the pairs represent equivalent ratios 8:5 = 16:10 etc. • y = 5/8(x).	» If students have difficulty ask them "what is the ratio of the y to the x-coordinate of each point?"	» Are students seeing that graphs of pairs of equivalent ratios form straight lines passing through the origin and are in the form $y = mx$ – the slope being the unit rate?

Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Superimpose rectangle H (ratio of sides = 2:5) with one corner aligned with the other rectangles (ratio of sides = 2:5) – what do you notice? » Can you explain why this is so as 5 - 3 = 8 - 5?	• The opposite corner of the rectangle does not line up because the sides are in a different ratio (2:5) and only equivalent ratios will lie on the same line through (0, 0). • The factor of change, not the difference between the numbers, determines the ratio.	» Give students a few minutes to discuss with the person beside them so that they can verbalise their thinking. » For homework, ask students to investigate the ratio of volume to surface area for cubes of side n where $1 \leq n \leq 9$, $n \in N$	» Can students see that 5:8 is a different ratio to 2:5? If they are applying additive reasoning they might incorrectly say that they were equal because the corresponding differences 8 - 5 and 5 - 2 are both 3.

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr><td colspan="4">Section H: Introduction of rate – comparing two quantities by division, which have different units</td></tr>
<tr>
<td>» A ratio can also be a rate where we compare quantities by division with different units for each quantity e.g. speed as the distance travelled per unit time e.g. km/hr

» In pairs, can you come up with other examples of rates?</td>
<td>• kms per litre.

• €5 per kg, 2 for €1, 40c each, €1.75 per dozen (and other price rates).

• 50 words per minute.

• People per km^2 (population density).

• Call charges – euro per minute.

• Passengers per car/bus/plane.

• Luggage weight per passenger etc.</td>
<td>» Where students have difficulty, ask them to think in terms of sport, shopping, travel and remind them that the word "per" is often used when dealing with rates.</td>
<td>» Can students see the difference and the similarity between ratio and rate – the same basic idea but rates involve units?

» Are students able to come up with several different examples of rates?</td>
</tr>
<tr>
<td>» If Shop A sells 5kg of pears for €12 and Shop B sells 4kg of pears for €11, which is better value for money?

» Are the rates the same? Why?

» Discuss for a minute in pairs how to compare the prices. Put the data into a table.

» You have found unit rate which is a very useful way of comparing rates and also of finding other prices.

» What will 7kg of pears cost in Shop A?</td>
<td>• You see that the rates are different, which you see when you find the price for 1kg.
<table>
<tr><td rowspan="3">Shop A</td><td>mass/kg</td><td>price/€</td></tr>
<tr><td>5</td><td>12</td></tr>
<tr><td>1</td><td></td></tr>
<tr><td rowspan="3">Shop B</td><td>mass/kg</td><td>price/€</td></tr>
<tr><td>4</td><td>11</td></tr>
<tr><td>1</td><td>?</td></tr>
</table>

• €2.40 per kg in Shop A

• €2.75 per kg in Shop B

• 7kg cost 7 x €2.40 = €16.80</td>
<td>» Write the example on the board.

» Ask different pairs of students for their opinions to ensure that students do not think they are the same rate. If they do, ask the class if this can be justified and allow the students to confront and explain the misconception.</td>
<td>» Do some students think the rates are the same as when the weight increases by 1 unit i.e. 1kg, the price increases by 1 unit i.e. €1?</td>
</tr>
</table>

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section I: Direct Proportion v Inverse Proportion			
» We are dealing again with equal ratios in the prices of pears for Shop A, 5 pears:€12 = 1pear:€2.40 = 7 pears:€16.80 » When we have statements of equal ratios, we have a proportional situation. We sometimes call it "direct proportion". » If it takes 5 painters 4 days to paint a building, how long will it take 1 painter working at the same rate to paint the building? » This is what is called **inverse proportion**. You will meet it again later on (Section N) in ratio and proportion and also when we investigate different patterns in mathematics.	$\frac{5}{12} = \frac{1}{2.4} = \frac{7}{16.8}$ • It will take 1 painter 20 days. 5:4 = 1:4/5 and is not equal to 1:20. • This is a different type of situation. It will take 1 painter 5 times longer than it takes 5 painters.	» Remind students to look at the context of the question and decide if it is appropriate to use equivalent ratios.	» Do students see the importance of the context of the question?
» Can you think of situations where keeping proportions correct is important?	• The taste and texture in baking depends on the proportions of ingredients. If you reduce quantities you must keep the same proportions. • Pharmacists making up drugs. • Making perfume. • Mixing cement, pricing items, architects drawing plans to scale. • Work-life balance. • Enlarging photographs.		» Are students coming up with several examples of situations where keeping quantities in proportion is important?

KEY: » next step • student answer/response

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» I am making pastry. I need twice as much flour as I do butter. Express this as a ratio.	• Flour:Butter = 2:1		
» I only need a small amount of pastry and I only have 110g of flour. How much butter will I need?	• I need ½ the amount of butter i.e. 55g of butter.		
» Write these values as a ratio equivalent to the 2:1 ratio.	• Flour to butter is 2:1 = ¼:⅛	» Write on the board: Flour: Butter = 2:1 = ¼:?	

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Teacher Reflections

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<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr><td colspan="4">Section J: Multiplicative v Additive relationships</td></tr>
<tr>
<td>» At Dave's party there were 2 cakes for every 3 people. At Ray's party there were 3 cakes for every 5 people. Does this tell us how many people were at each party?

» What does it tell us?

» Will they get more or less than ½ of a cake each at each party?

» Did Dave's guests or Ray's guests have more cake to eat?</td>
<td>• No. We do not know how many were at each party.

• It tells us that if we had 2 cakes for 3 people this means that if 6 people attended then we would need 4 cakes – double the people then double the cakes.

• We know they will get a bit more than ½ each as we have 2 cakes (4 halves) among 3 people and 3 cakes (6 halves) for 5 people.

• Dave's party:
3 people: 2 cakes
1 person: 2/3 cake

• Ray's party:
5 people: 3 cakes
1 person: 3/5 cake

• 2/3 > 3/5 so Dave's guests get more cake!</td>
<td>» Encourage students to tabulate information to help clarify the situation.

» Remind them that it is easier if the unknown of the 4 quantities involved is put last in the table.

» Draw tables on the board where students have difficulty organising this by themselves. Remind students to label the columns so as not to confuse figures.

Dave's party:
<table>
<tr><th>No. of people</th><th>No. of cakes</th></tr>
<tr><td>3</td><td>2</td></tr>
<tr><td>1</td><td>?</td></tr>
</table>
Ray's party:
<table>
<tr><th>No. of people</th><th>No. of cakes</th></tr>
<tr><td>5</td><td>3</td></tr>
<tr><td>1</td><td>?</td></tr>
</table>
» See Appendix 1 for a pictorial representation of this using partitioning as a possible additional activity.</td>
<td>» Do students see that this is multiplicative and not additive reasoning? 2 cakes for 3 people does not mean that if we have 6 people (+3 people) that we will have +3 cakes.

» Did students reverse the order in which the quantities were given to them to make it easier to make the comparison?</td>
</tr>
</table>

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section K: Is it always necessary to use unit rate?			
» If 2 apples cost 60c, what will 4 apples cost given the same price rate? Is there more than 1 way of finding the answer? » What you noticed here was the "factor of change" for the apples and applied the same "factor of change" for the prices.	• 1 apple costs 30c, hence 4 apples cost €1.20 • 4 apples will cost twice as much as 2 apples, hence €1.20	» Draw a table on the board showing answers. Apples | € 2 | 0.60 4 | 1.20	» Were students able to apply a factor of change method as well as unitary method?
» What equivalent ratios can you see in this table? » Instead of using numbers express the ratios in terms of the names of the measures used.	• 2:4 = 0.60:1.20 • 4:2 = 1.20:0.60 • 2:0.60 = 4:1.20 • 0.60:2=1.20:4 • apples:apples = price:price • apples:price = apples:price	» Show the equivalent ratios using arrows on the table. Apples | € 2 | 0.60 4 | 1.20 » Encourage students to put information into a table.	» Can students see that using a table can clarify the information and do they see the importance of comparing corresponding quantities in the correct order?

<table>
<tr>
<th>Student Learning Tasks: Teacher Input</th>
<th>Student Activities: Possible Responses</th>
<th>Teacher's Support and Actions</th>
<th>Assessing the Learning</th>
</tr>
<tr>
<td colspan="4">Section L: Using both methods (unit rate and factor of change) where the factor of change is not an integer</td>
</tr>
<tr>
<td>» Maeve bought 3kg of apples for €6.90. What would 10kg cost her at the same price rating?

» Work in pairs and use more than 1 method to solve this.

» Which method is most efficient/user friendly and why do you think so?</td>
<td>
<table>
<tr><th>Apples/kg</th><th>Price in €</th></tr>
<tr><td>3</td><td>6.90</td></tr>
<tr><td>1</td><td>?</td></tr>
<tr><td>10</td><td>?</td></tr>
</table>

1kg would cost €6.90/3

• 10kg would cost €6.90/3 x10 = €23.00

• or 9kg costs 3x €6.90 = €20.70 and 1kg costs €2.30

• Hence 10kg costs €23.00

• The unit rate method is easier than using factor of change because 10 is not an integer multiple of 3. If I had been asked to work out a price for a multiple of three, I would use the factor of change method.</td>
<td>» Circulate and check that students are tabulating information and labelling the columns.</td>
<td>» Are students able to evaluate different methods and see that while different methods can yield the correct result, it is a case of "horses for courses" in maths as well as at the races?</td>
</tr>
</table>

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Teacher Reflections

<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr>
<td>» Ham is being sold at €10.50 for 2kg. The butcher is selling a piece weighing 13.14kg. What will this cost using the same price rate? You are not to use a calculator. Break down the numbers so that you can use easy multiplications and divisions.</td>
<td>
<table>
<tr><th></th><th>Kg</th><th>Cost</th><th>Operation</th></tr>
<tr><td>A</td><td>2</td><td>10.50</td><td></td></tr>
<tr><td>B</td><td>1</td><td>5.25</td><td>A/2</td></tr>
<tr><td>C</td><td>10</td><td>52.5</td><td>Bx10</td></tr>
<tr><td>D</td><td>0.1</td><td>0.525</td><td>B/10</td></tr>
<tr><td>E</td><td>3</td><td>15.75</td><td>B/2</td></tr>
<tr><td>F</td><td>13.1</td><td>68.775</td><td>C+D+E</td></tr>
<tr><td>G</td><td>0.01</td><td>0.0525</td><td>D/10</td></tr>
<tr><td>H</td><td>0.04</td><td>0.21</td><td>Gx4</td></tr>
<tr><td>I</td><td>13.14</td><td>68.985</td><td>F+H</td></tr>
</table>
</td>
<td>» If necessary lead students through the process step by step by asking questions corresponding to the lines in the table.</td>
<td>» Can students see that it is possible to work their way logically through a problem without the use of an algorithm or a calculator.

» Is this type of exercise giving students the skills and confidence to break problems down into simple steps?</td>
</tr>
<tr>
<td>» Mark and Mandy buy a bag of chocolate buttons each. Mark eats 5 buttons every 25 minutes and Mandy eats 3 buttons every 10 minutes.

» If there are the same amount of buttons in each bag and they keep eating until the bags are empty, who will finish first?

» In pairs, complete Section Student L: Activity 7.</td>
<td>
<table>
<tr><th>Name</th><th>Buttons</th><th>Time/ms/mins/h</th></tr>
<tr><td>Mark</td><td>5</td><td>25</td></tr>
<tr><td>Mandy</td><td>3</td><td>10</td></tr>
</table>
Unit Rate
• Mark: 1 button every 5 minutes
• Mandy 1 button every 3 1/3 minutes
• Mandy is eating her buttons faster and will finish first.
Making times equal
• After 5 minutes Mark will have eaten1 button
• After 5 minutes Mandy will have eaten 1.5 buttons on average. Mandy will finish first.
Making buttons eaten equal
• Mark: 15 buttons in 75 minutes Mandy: 15 buttons in 50 minutes so she finishes first.
</td>
<td>» If students are having difficulties, get them to make out a ratio table for Mark and another for Mandy. Given equal time intervals, how many buttons have been eaten by both parties or, given equal numbers of buttons, what times have elapsed? Use this information to make a decision on who is fastest.
• Mark
<table>
<tr><td>Buttons</td><td>5</td><td>10</td><td>15</td></tr>
<tr><td>Time</td><td>25</td><td>50</td><td>75</td></tr>
</table>
• Mandy
<table>
<tr><td>Buttons</td><td>3</td><td>6</td><td>9</td><td>12</td><td>15</td></tr>
<tr><td>Time</td><td>10</td><td>20</td><td>30</td><td>40</td><td>50</td></tr>
</table>
• Distribute Section L: Student Activity 7.
</td>
<td>» Can students make sense of different ways of solving this problem and justify the method they are using?</td>
</tr>
</table>

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<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher’s Support and Actions</th><th>Assessing the Learning</th></tr>
<tr><td colspan="4">Section M: Speed, distance and time</td></tr>
<tr><td>» Speed is a named ratio which is also a rate. In pairs discuss this and write down what you think this means.</td><td>• It compares two quantities by division so it is a ratio and the quantities have different units so it is also a rate.</td><td>» Write on the board: Speed – express as a ratio of two quantities, giving units.

» Elicit answers from different groups and check for class consensus.</td><td>» Can students link ratio and rate to the everyday concept of speed which they are all used to?</td></tr>
<tr><td>» Give examples of units of speed?</td><td>• km/h, m/s, miles/h</td><td>» Write the answers on the board.</td><td></td></tr>
<tr><td>» How do the units help in telling you how to calculate speed?</td><td>• They tell you it is the ratio km:h or km/h to get the unit rate. Knowing the units gives the order of the ratio.</td><td></td><td>» Do students realise that when asked to calculate a rate, the units are the key to the order of the ratio?</td></tr>
<tr><td>» A train travels 210km in 2 hrs. What is its speed in km/h?

» If we divided both numbers by 210 what information would this give us?</td><td>• We need to find out how many km the train travels in one hour?<table><tr><th>Time/h</th><th>Distance/km</th></tr><tr><td>2</td><td>210</td></tr><tr><td>1</td><td>?</td></tr></table>2h:210km = 1h:105km

• Dividing both numbers by 210 gives the time for 1km in hours i.e. h/km.</td><td></td><td></td></tr>
<tr><td>» What is 100km/h in m/s?</td><td>• 100 km/h = 100000m/h
100000/3600m/s =27.7m/s</td><td>» Encourage students to take this one step at a time.</td><td></td></tr>
</table>

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<table>
<tr><th>Student Learning Tasks: Teacher Input</th><th>Student Activities: Possible Responses</th><th>Teacher's Support and Actions</th><th>Assessing the Learning</th></tr>
<tr><td>» When we say that the speed of a car for a journey was 80km/h, does that mean that the speedometer was showing 80km/h for every instant of that journey?

» How is this calculated?</td><td>• No. Depending on road conditions and traffic the speed would vary around 80km/h which is the average speed.

• It is total distance travelled divided by total time taken.</td><td>» Refer students back to how they calculated, for instance, the mean mark for the class.</td><td>» Do students appreciate the difference between average speed and instantaneous speed?</td></tr>
<tr><td>» Paula decided to cycle to her friend's house 20km away. She cycled at 20km/hr and covered half the journey but then she got a puncture and had to walk for 2 hours to finish the journey. Find the average speed.

» Do Section M: Student Activity 8 in pairs.</td><td>• Average speed = total distance/ total time

• 20km/hr for 10km [km/(km/h) = h]

• time = distance /speed = 0.5h

• average speed = 20/2.5= 8 km/h</td><td>» Distribute Section M: Student Activity 8.</td><td>» Are students able to break down this problem into simple steps and use prior knowledge?</td></tr>
<tr><td>» Do you expect currency conversions to be proportional situations i.e. involving equivalent ratios? Why or why not?</td><td>• Yes because currency conversions involve rates and the rate is the same for each euro or denomination being converted.</td><td>» Show students where relevant information on currency conversions can be found e.g. www.xe.com</td><td></td></tr>
<tr><td>» €1 will buy you £0.83. If I buy €500 worth of sterling, how many pounds sterling will I get for my €500? Work this out using what you have learned about ratio and proportion.</td><td><table><tr><td>euro</td><td>Sterling</td></tr><tr><td>1</td><td>£0.83</td></tr><tr><td>€500</td><td>?</td></tr></table>
• €1: £0.83 = €500: £?

• You form equivalent ratios – multiply 1 and 0.83 by 500. 0.85 x 500 = £415</td><td>» Check that students are setting out the information in a table first.</td><td>» Are students applying equivalent ratio ideas to currency conversions?</td></tr>
</table>

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» I did not spend all of the sterling I bought and have £153 left. I wish to change it back to euro and the exchange rate has not changed. How many euro will I get for it if there is no charge for the exchange? » If I want to know what €153 is worth – what do I need to know? » Do **Section M: Student Activity 9** in pairs.	<table><tr><td>Sterling</td><td>Euro</td></tr><tr><td>0.83</td><td>1</td></tr><tr><td>1</td><td>?</td></tr><tr><td>153</td><td>?</td></tr></table> • We need to know what £1 is worth. • £0.83 = €1 • £1 = € $\frac{1}{0.83}$ (Divide both sides in the second row by 0.83) • £153 = 153 x € $\frac{1}{0.83}$ = €184.34	» This is more difficult as we don't know the unit rate and the figures don't fit nicely with a factor of change method. » Distribute **Section M: Student Activity 9.**	» Do students see that the figures do not fit nicely with a factor of change method and that unit rate is the best method in this case?

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<tr><td colspan="4">Section N: Inverse Proportion (See also Section I)</td></tr>
<tr><td>» Situations like currency exchange involve equal ratios i.e. proportions. When one of a pair of related quantities increased/ decreased the other related quantity increased/ decreased in the same proportion. Can you think of any two related quantities where when one gets bigger the other gets smaller?</td><td>• The more people who share the lotto prize money, the less each person receives.
• If we book a bus for a school trip at a fixed price, the more people who go on the trip the smaller the fare will be per person.
• Doubling the speed to travel a fixed distance halves the time.</td><td>» Refer to proverbs perhaps as in “Many hands make light work”.
» Put students into small groups to brainstorm this activity.</td><td>» Can students list many different examples of inverse proportions?</td></tr>
<tr><td></td><td>• In science, we found that for a fixed mass of gas at constant temperature, as the pressure increased the volume decreased proportionally. This is Boyle’s Law.</td><td></td><td></td></tr>
<tr><td>» For each situation (e.g. "school trip" and "travelling a fixed distance") can you identify which quantity is constant and which quantities can vary?
» What is the relationship between the constant quantity and the varying quantities?</td><td><table><tr><th></th><th>School trip</th><th>Travelling a fixed distance</th></tr><tr><td>Constant</td><td>Bus price (p)</td><td>Distance (d)</td></tr><tr><td>Varying</td><td>No. of persons (n)</td><td>Speed (v)</td></tr><tr><td>Varying</td><td>Cost per person (c)</td><td>Time (t)</td></tr><tr><td>Relationship</td><td>$p = n \times c$</td><td>$d = v \times t$</td></tr></table></td><td>» Different groups should use their own examples and look for a class consensus for each example.
» Check that students have completed at least 4 examples showing the relationship between the constant and the variables.</td><td>» Can students use reasoning to work out the relationship rather than relying on a formula as in distance = speed x time?</td></tr>
</table>

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Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section E: The ratio of consecutive numbers in the Fibonacci sequence			
» What do you notice about the relationship between the constant and the varying quantities?	• The constant is always the same as the product of the varying quantities.	» Encourage students to look for patterns and links between different situations.	» Are students able to move from the particular to the general?
» We call relationships like these **inverse proportions**. If k is the constant and x and y are the varying quantities, can you generalise the relationship between k, x and y?	• $k = xy$	» For OL classes it is not necessary to generalise formally using symbols.	» Are some students able to generalise using symbols?
» For the school trip, if the cost of the bus is €100 and if 20 people travel how much does each pay? What is the cost per person if 40 people travel? » How is this different from the following question? If 20 books cost €100, what will 40 books cost at the same cost per book? » In pairs come up with similar examples.	• If 20 students travel the cost per student is €5. If 40 students travel the cost per person is halved i.e. €2.50. • The second situation involves equal ratios. 20:100 = 40:200. The costs are in proportion. • The more bars I buy the more it will cost me. However if I have a fixed amount to spend then the cheaper each bar is the more bars I can buy.	» Remind students that in inverse proportions, one quantity is fixed e.g. fixed amount of prize money, fixed mass of gas, fixed length of wood (the longer the pieces cut from it the less of them you can cut.)	» Are students able to make up their own examples of where quantities are related proportionally and in inverse proportion?

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
» Complete the following activity in pairs. A building job takes 40 days to complete if 10 people work on it. How many days will it take to complete if 25 people work on it working at the same rate? » Predict first whether the answer will be more or less than 10 days. » Does this involve a proportional situation? Explain. » Complete **Section N: Student Activity 10** in pairs. Not all of the questions will involve inverse proportions.	• The answer will be less than 40 days because more people will take less time. • 1 person: 40 x 10 = 400 work days. • 25 people: 400/25 = 16 work days. or • number of people (n) and number of days (d) taken for different numbers of people can vary. $n_1d_1=n_2d_2$ 40 x 10 = 25 x d_2 d_2 = (40 x10)/25 This does not involve equal ratios. 40:10 ≠ 25:16 40:10 = 4:1 25:16 = 4.53:1	» Explore the second approach with HL classes. » Distribute **Section N: Student Activity 10.** » Give **Section O: Student Activity 11** to students who finish Student Activity 10 early.	» Are students making predictions before carrying out calculations and using their prediction to come up with a strategy?

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Teacher Reflections

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Teacher Reflections

Student Learning Tasks: Teacher Input	Student Activities: Possible Responses	Teacher's Support and Actions	Assessing the Learning
Section O: Two rates compared			
» Alice can paint a room in 3 hours. Pat can paint the same room in 1.5 hours. Working together, how long will it take the two of them to paint the room? » Will the two of them together paint the room in more or less time than when working on their own? » Do **Section O: Student Activity 11** in pairs. » When you have finished writing Q6 and Q7, and after you have worked out a solution, swap your Q6 and Q7 with the group beside you without swopping solutions.	• It will take less time working together. • If we made the **times equal**, we could see how many such rooms could be painted by each of them in equal times. • The Lowest Common Multiple (LCM) of the times is 3 hours. In this time Alice could paint 1 room whereas in 3 hours Pat can paint 2 rooms. • Working together the 2 of them can paint 3 rooms in 3 hours. Hence they could paint one room in 1 hour. • Or we could work out how much of a room is painted by each in 1 hour i.e. $\frac{1}{3} + \frac{2}{3} = 1$ room in 1 hour.	» Remind students that when comparing they must compare like with like even in Maths! » Encourage students to make out a table for each situation to clarify their thinking, » Distribute **Section O: Student Activity 11** » Circulate and check progress and the type of questions students are writing for Q6 and Q7. » Check to see if students are able to solve each other's questions and if not, encourage dialogue between the groups in the form of hints without giving the solution away. » Share the questions with the class at the end of the session for homework.	» Are students able to figure out from prior knowledge how to tackle these questions on combined rates by making the times equal?

Section A: Student Activity 1

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1. Which class has more girls? Which class has the greater proportion of girls?

Class	2A	2B
Number of students in the class	20	26
Number of girls	10	11

2. You are a Manchester Utd. fan. Which class, 1A or 1B, would you prefer to be in? Explain.

	Man Utd fans	Liverpool fans	Total no. of students in class
Class 1A	10	20	30
Class 1B	9	9	18

3. Simplify the following ratios

   a) 9:15

   b) 60:84

   c) 0.25:0.8

   d) $\frac{5}{2}:\frac{5}{4}$

   e) $4:\frac{5}{8}$

   f) $3\frac{1}{2}:1.4$

   g) 0.4kg:500g

4. Six students in a class of 32 are absent. Find the ratio of

   a) The number absent to the number present

   b) The number absent to the total number of students in the class

   c) The number present to the total number of students in the class

Which ratios could be written as fractions? Explain your answer.

5. Two baby snakes are measured in the zoo. One measures 8cm and the other measures 12cm. Two weeks later they are measured again and the first one now measures 11cm while the second one measures 15cm. Did the snakes grow in proportion? Explain your answer.

Section B: Student Activity 2

1. Divide €20 between Patrick and Sarah in the ratio 3:2
2. Divide €20 between Patrick and Sarah in the ratio 4:3. Give your answer to the nearest cent.
3. A sum of money is divided between Laura and Joan in the ratio 2:3. Laura gets €8. How much did Joan get?
4. Joe earns €3,500 per month. The ratio of the amount he saves to the amount he spends is 2:5. How much does he spend? Work out how much he saves in two different ways.
5. The total area of a site is 575m². Anne is building a house on the site. The ratio of house area to garden area is 5:8. Find the area of the garden.
6. Julianne and Kevin inherit €5,500. Julianne is 24 and Kevin is 9. The money is to be divided in the ratio of their ages. How much will each receive?
7. In class A, $\frac{3}{4}$ of the class watch American Idol. In class B, $\frac{9}{10}$ of the class watch American Idol. What is the ratio of the students who do not watch American Idol in class A to those who do not watch American Idol in class B?
8. In a 1,000ml mixture of fruit concentrate and water there is 40ml of fruit concentrate.
   a. What is the ratio of the amount of fruit concentrate to the amount of water?
   b. If 200ml of concentrate is added to the mixture, what is the ratio of the amount of fruit concentrate to the total volume of mixture.
   c. What is the ratio of concentrate to water in the new mixture?
9. Two containers, one large and one small, contain a total of 4 kilograms of bath salts. One quarter of the bath salts from the large container is transferred to the small container so that the ratio of bath salts in the large container to that in the small one becomes 3:2. How many kilograms of bath salts were originally in each container?

Section C: Student Activity 3

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1. The heights of Derek, Alan and Jim are 1.7m, 180cm and 150cm. Find the ratio of the heights of Derek, Alan and Jim.
2. €45 is divided among 4 children in the ratio 1:2:3:9. How much did each child receive?
3. An apartment has an area of 47m². It is divided into living, sleeping and dining areas in the ratio of $\frac{1}{5}:\frac{1}{4}:\frac{1}{3}$.

   What is the area of the smallest section?
4. If a:b:c = 5:10:9, divide 3700 in the ratio $\frac{1}{a}:\frac{1}{b}:\frac{1}{c}$
5. €144 is divided between Jean, Alice and Kevin. Jean gets half as much as Alice and one third as much as Kevin. How much does each of them get?
6. Write a similar question to Q1 and solve.
7. Write a similar question to Q2 and solve.
8. Write a similar question to Q3 and solve.
9. Write a similar question to Q4 and solve.
10. Write a similar question to Q5 and solve.

Section D: Student Activity 4

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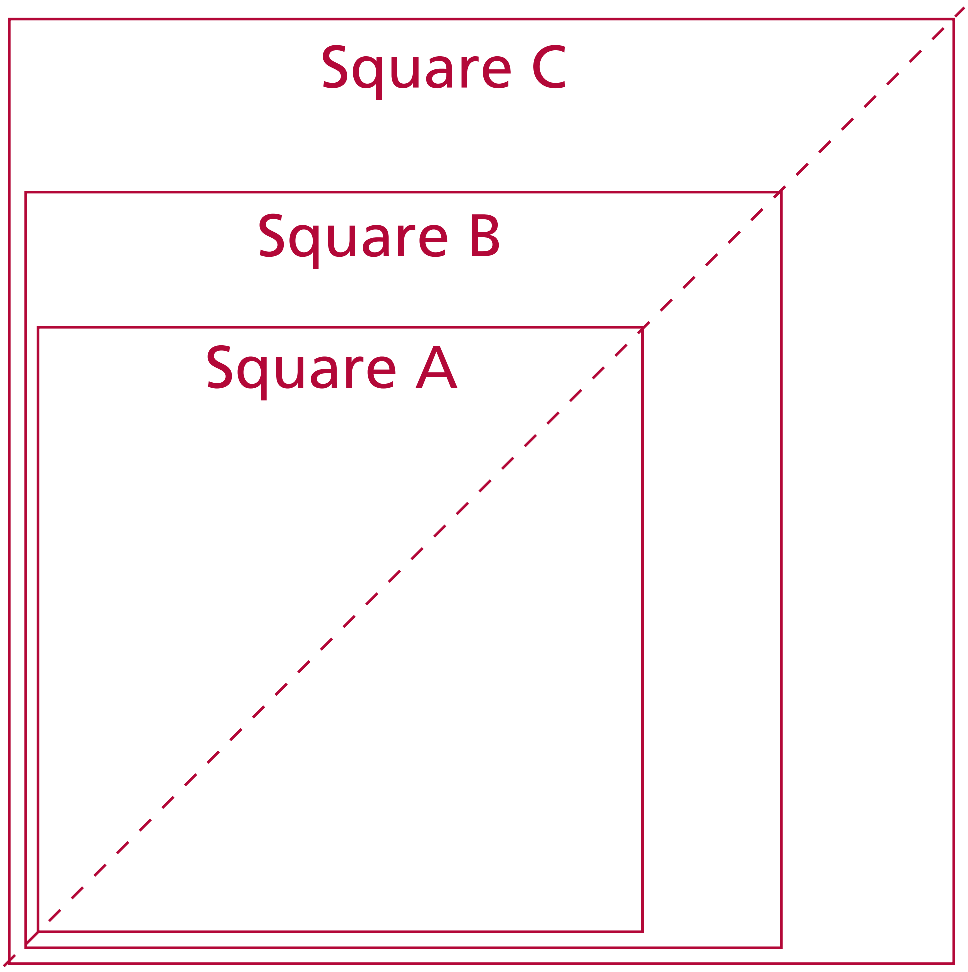
Measure the length of the side of each square in mm and the length of the diagonal in mm. Find the ratio of the length of the diagonal to the length of the side and write the equivalent ratio in the form x:1 where x is written correct to one decimal place.

	Length of the diagonal/mm	Length of the side/mm	Length of the diagonal: Length of side $=x$:1
Square A			
Square B			
Square C			
My Square (Draw 3 squares of different side lengths, all of which are bigger than square C. Measure the ratio of diagonal length/side length.)			

What pattern have you observed?



Square A
Diagonal length _____mm
Side length _____mm

Square B
Diagonal length _____mm
Side length _____mm

Square C
Diagonal length _____mm
Side length _____mm

Section D: Student Activity 4

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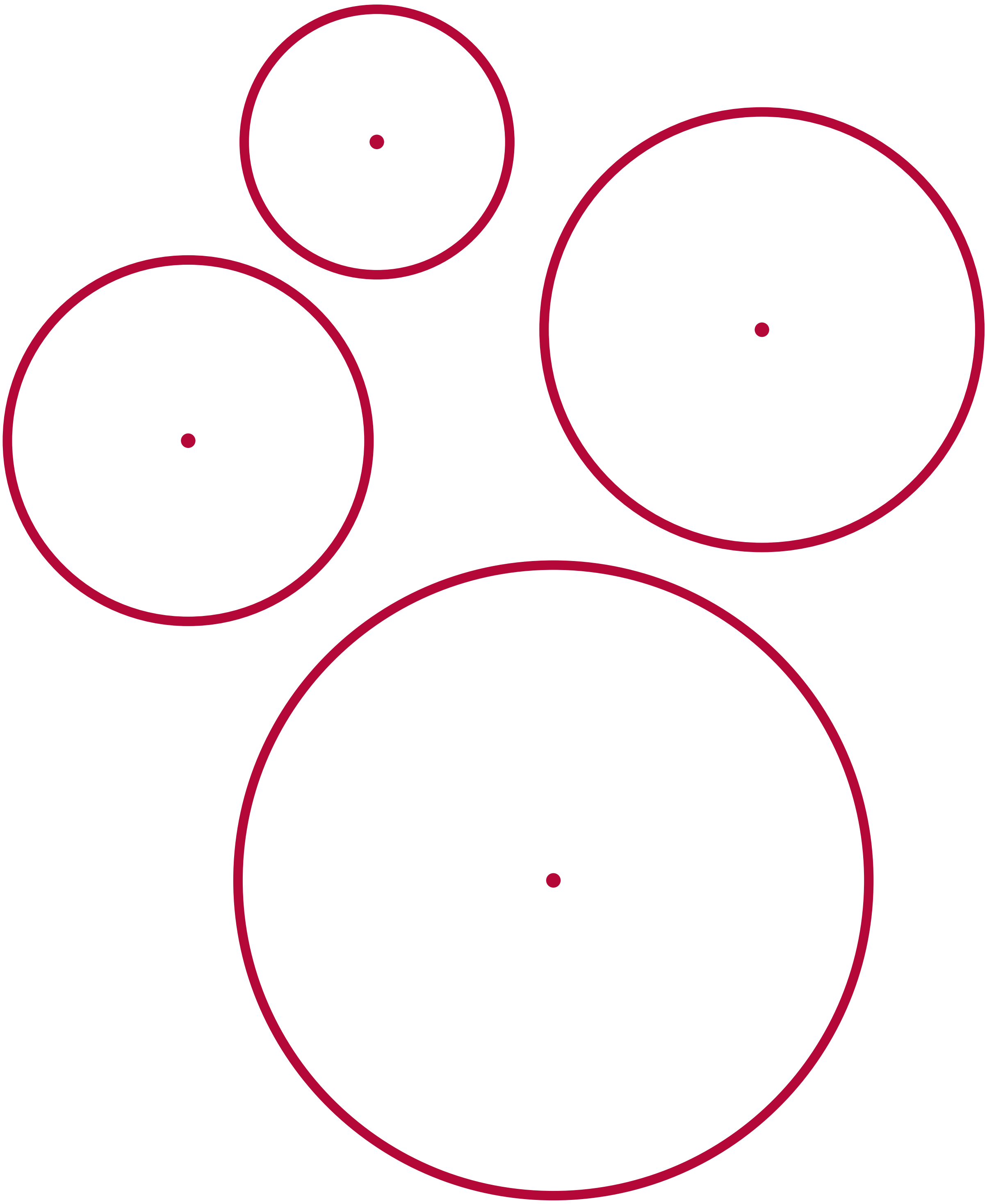
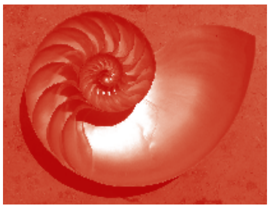
Measure the lengths of the diameters and circumferences of the following circles to the nearest millimetre. Find the ratio of the length of the circumference to the length of the diameter. What do you notice? Draw another circle with a different diameter and again find the ratio of circumference to diameter. What do you notice? Put findings into a table of values.



Section F: Student Activity 5



The nautilus starts out as a very small animal, which only needs a very small shell. As it grows it needs a bigger shell. Using squared paper, we can model a spiral growth using a square of side 1 unit in which we draw a ¼ circle representing the growth of the spiral.

As the spiral grows, draw another square of side 1 on top of the first one and draw ¼ of a circle in this, continuing on from the first quarter circle.

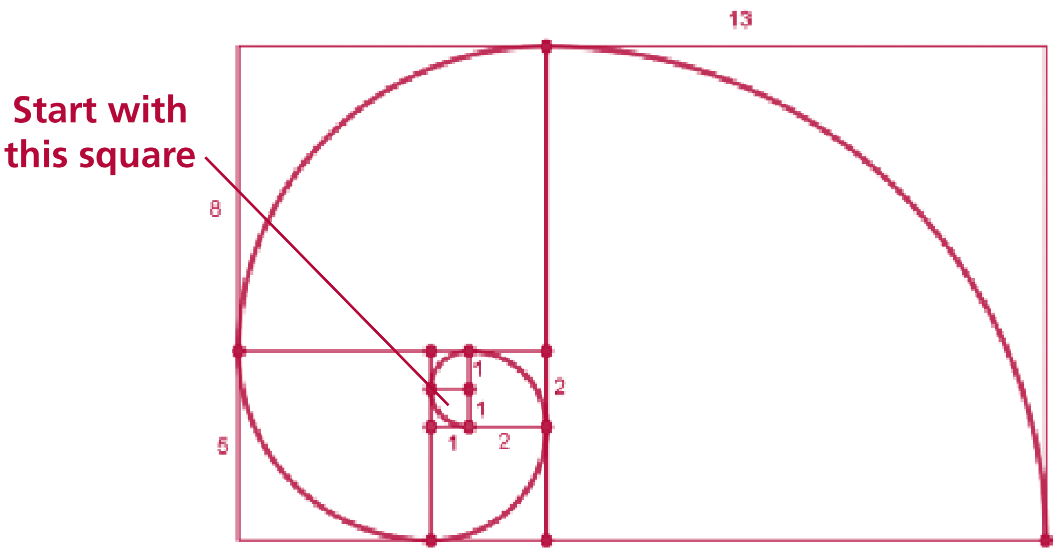
For the next stage of growth draw a square of side 2 beside the 2 small squares and draw a quarter circle in this square to represent the continued growth of the spiral.

Next draw a square of side 3 onto the rectangle formed by the first 3 squares and again draw a quarter circle in this square to show the continued growth of the spiral.

Then, a square of side 5 is annexed onto the 4 squares, (which forms a rectangle), and again the quarter circle is drawn to represent the spiral growth.

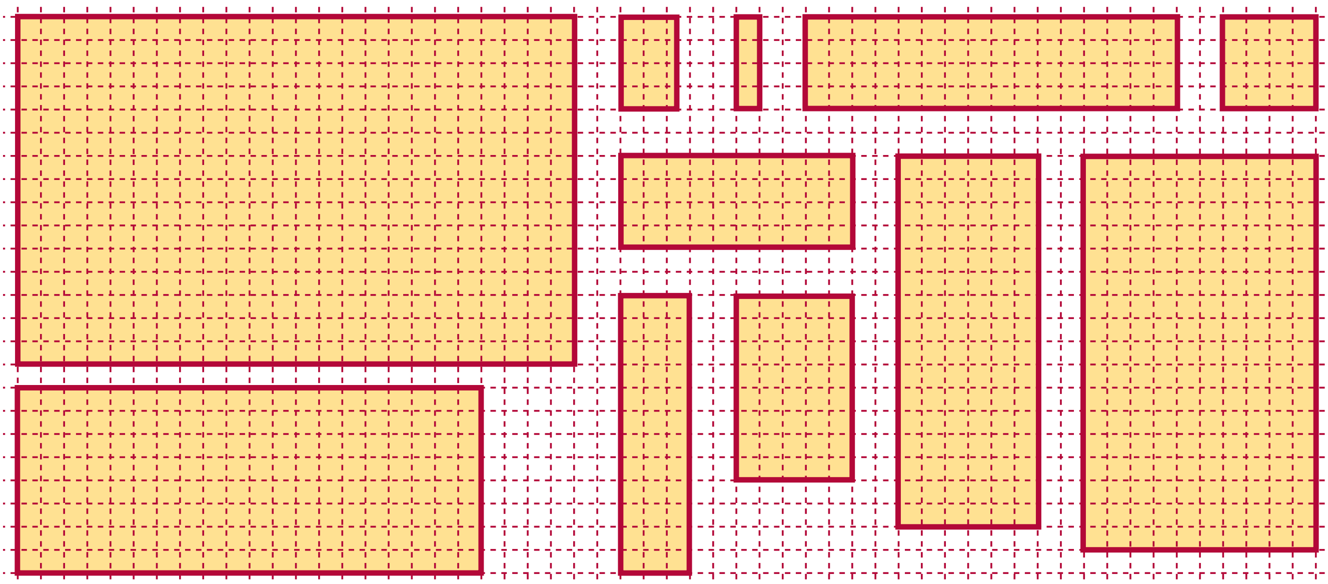
This pattern carries on, continuously annexing a square of side equal to the longer side of the last rectangle formed, (the sum of the sides of the last two squares formed), and filling a quarter circle into this square.

The spiral shape which we get is not a true spiral as it is made up of fragments which are parts of circles but it is a good approximation of spirals which are often seen in nature, like that of the nautilus shell.



Section G: Student Activity 6

Which rectangles look-alike or are similar? Make a guess first and write down your guesses at the bottom of the page. Then justify by taking measurements of lengths of sides of each rectangle; short side (S) first, then longer side (L). (Use the length of a unit square on the grid as a unit of measurement.) Are there any odd ones out?



	A	B	C	D	E	F	G	H	I	J	K
S											
L											
S:L											
S:L (simplified)											

My guesses on which rectangles look-alike:____________________________

Having filled in the table above, which rectangles do I now think look-alike or are similar?

Put them into groups and say why you put them into these groups.

Group 1: _______________________ Why? ____

Group 2: _______________________ Why? ____

Group 3: _______________________ Why? ____

Section L: Student Activity 7

1. In which of the following examples is the ratio of the number of apples to the number of cents the same? How will you compare them?

🍎🍎🍎 A. €0.90	🍎🍎 C. €0.60	🍎🍎🍎🍎 E. €1.20
🍎🍎 B. €0.80	🍎 D. €0.30	🍎🍎🍎 F. €1.20

Make out tables for the following questions to show the information given.

2. A shop is selling 4 apples for €2.
   a) How much does one apple cost? b) How many apples will I get for €1?
   c) How much will 7 apples cost?
3. If copies cost €5 for 10 copies, what will 25 copies cost? Work this out by unit rate and by another method.
4. Luke bought 10 pens for €2.40. What would 12 pens cost him at the same rate? Show two methods of working out the answer.
5. Five out of every eight students in a local college are living away from home. Of the 200 students studying mathematics in the college, how many will be living away from home if the same ratio is maintained?
6. Elaine can cycle 4km in 14.7 minutes. How far can she cycle in 23 minutes if she keeps cycling at the same rate?
7. The Arts Council is funding a new theatre for the town. They have a scaled down model on show in the local library. The building is rectangular in shape. The dimensions of the model are 1m x 0.75m. If the longer side of the actual building is to be 40m, what will be the length of the shorter side? What is the ratio of the floor area of the actual building to the floor area of the model?
8. Aidan cycles 4km in 12 minutes. Karen can cycle 2km in 5 minutes. Which of them is cycling fastest? Explain. Give at least three different methods for finding the answer.
9. Which is the stronger coffee or do any of the following represent the same strength coffee?
   a) 3 scoops of coffee added to 12 litres of water
   b) 2 scoops of coffee added to 8 litres of water
   c) 4 scoops of coffee added to 13 litres of water

Section M: Student Activity 8

Project Maths Tionscadal Mata Development Team

1. Find the value of x in the following proportions:

   a) $8:3 = 24:x$ b) $9:2 = x:12$ c) $3:x = 15:10$

   d) $x:6 = 6:18$ e) $9:12 = 6:x$ f) $3:2 = (x+5):x$

2. Eight cows graze a field of 2 hectares. Express this rate as hectares/cow. Express it as cows/hectare.

3. The average fuel consumption of Joe's Audi A1 1.4 TF SI is approximately 18 km/litre. Joe fills the tank with 40 litres of fuel. How far can Joe travel before the tank is empty? The average CO_2 emission rate is 126 g/km. How many grams of CO_2 are emitted by the car in consuming 40 litres of fuel?

4. If I travel 700km in 3.5 hours, express this as a rate in:

   a) kilometres per hour b) kilometres per second c) metres per second

5. The speed of sound is approximately 340 m/s. Express this speed in km/hour.

6. Usain Bolt won the 200m in the Beijing Olympics in 19.30 seconds breaking Michael Johnson's 1996 record by two hundredths of a second. What was his average speed in km/hour? What was Johnson's average speed in km/hour?

7. The legendary American racehorse Seabiscuit completed the 1.1875 mile long course at Pimlico Race Track in 1 minute 56.6 seconds. This was a track record at the time. What was his average speed in miles/hour correct to one decimal place?

8. Marian drives at a speed of 80km/hr for ¾ of an hour and at 100 km/hour for 1.5 hours.

   a) What distance has she travelled in total?

   b) What is her average speed in km/hour?

9. Derek took half an hour to drive from Tullamore to Athlone. His average speed for the entire journey was 80km/hour. If his speed for 1/5 of the journey was 120 km/hour, find:

   a) the time taken to drive this section of the journey

   b) what his speed was, in km/hour, to the nearest whole number, for the remainder of the journey if it was completed at a constant speed.

Section M: Student Activity 8

Project Maths
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Development Team

Homework/Class work

1. Patrick ran 4 laps of a track in 10 minutes. Ian ran 8 laps in 21 minutes. Who ran fastest? Explain.

2. Compare the performance of 2 players, David and Andy. David scored 20 goals out of 40 shots at goal whereas Andy scored 25 goals out of 50 shots at goal.

3. In the car park there are 12 silver cars and 8 blue cars.

   a) What is the ratio of silver cars to blue cars in its simplest terms?

   b) What is the ratio of blue cars to silver cars in its simplest terms?

4. Which is the better deal: €18 for 3 bracelets or €30 for 5 bracelets?

5. Which is the better deal: 4 hours worked for €12 or 7 hours worked for €28? Discuss.

Section M: Student Activity 9

For currency conversion rates see: www.xe.com

1. **Jean has a sister in America whom she planned to visit in May 2010. She had been watching the currency markets from July 2009 to May 2010 to decide when to buy US dollars (US$) for her trip.**

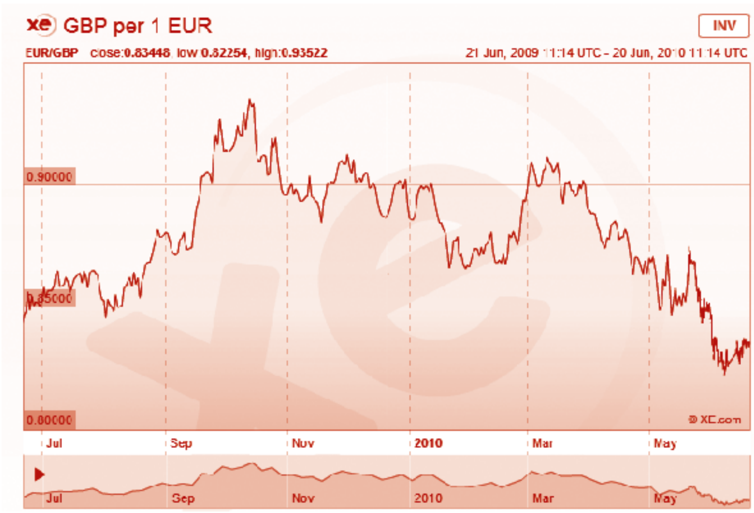
   a) **When should she have bought dollars to give her the maximum amount of dollars for her euro in this period. Using the above chart, for reference http://www.xe.com/currencycharts/?from=EUR&to=USD&view=1Y**

   b) **Estimate from the chart the exchange rate at that time in US$/€.**

   c) **How many US$ would she have got for €500 at that exchange rate?**

   d) **Jean returned to Ireland in mid June 2010. Looking at the trend in the above chart do you think it was wise for her to change her leftover dollars to euro immediately? Explain.**

2. **Some shoppers in the south of Ireland decided to shop in Northern Ireland due to changes in the currency exchange rate between the euro and pound sterling. Using the chart below, can you decide when they would have benefited most from this decision? Justify your answer.**



Section M: Student Activity 9 (continued)

3. Tom works for Agrimachines in Carlow. He has made a deal with a customer in Northern Ireland to sell him a machine for €15,000. At the time the deal was made the euro was worth £0.89. When the customer was paying a short time later the euro had fallen from £0.89 to £0.83. Was this good news for the customer? Justify your answer by calculating how much he would have paid in pounds sterling at both exchange rates.

4. Without using a calculator determine how much a machine costing €10,000 in Carlow would cost a customer in Northern Ireland when the exchange rate is £0.83/€. How much would a machine costing €5,000 in Carlow cost in Northern Ireland at the same exchange rate? Hence, how many £ sterling correspond to €15,000?

5. Colette is a currency trader for National Bank. Each day she buys and sells currencies in order to make a profit. In order to trade profitably she needs to know the conversion rates for several currencies. Can you help her answer some of the questions she has to deal with on a particular day? (If you wish, you can insert the actual conversion rates on the day you are doing these questions by checking on www.xe.com or any other relevant site or source of information.)

   a) Colette knows that on a particular day €1 buys 112.177 Japanese Yen. How much would it cost Colette's bank to buy 1,000,000 Yen? Use a table to show the information.

   b) The Thailand Baht (THB) is at a low of 0.38 Mexican Pesos (MXN). How many THB can Colette purchase for 1,000,000 MXN?

   c) Colette knows that one euro will buy 1.42 Australian Dollars (AUD). How many euros will 1AUD buy? The Australian Dollar has dropped from 1.79 AUD/€ to 1.42 AUD /€. Is this a good time for Colette to trade AUD for euro or euro for AUD? Justify your answer.

   d) €1 will buy 57 Indian Rupees (INR). €1 will buy US$1.24 (USD). How many INR will one USD buy? Before calculating, predict whether it will be more or less than 57 INR.

   e) Colette went on a trip to London. While shopping there, she saw a pair of shoes identical to a pair she had bought in Dublin for €88. In the London shop they cost £76. She knew that the exchange rate was £0.83/€. Which city was giving the better deal on the shoes? Explain.

   f) From London Colette was due to visit Switzerland and then Norway. She decided to cancel her trip to Norway and changed the 1,000 Norwegian Kroner (NOK) she had for Swiss Francs (CHF). At the currency exchange she received 174.22 CHF for her 1,000 NOK (no commission charge included). How many Norwegian Kroner did 1 Swiss Franc buy?

   g) Her hotel room in Zurich cost her 179 Swiss Francs per night. How much is this in euro if the exchange rate was €1 = 1.23 Swiss Francs? Predict first whether the answer in euro will be greater than or less than €179.

Section M: Student Activity 10

Project Maths
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1. It takes 30 people 60 working days to build a small bridge. How many people are needed if the bridge is to be built in 40 working days, assuming that they all work at the same rate. First decide whether your answer will be more or less than 30. How long would it take one person to build the bridge if it were possible for them to do it alone?

2. Using the scale given on the map to the right, work out the distance between Kilbeggan and Athlone in both km and miles. Use this to find the approximate ratio of miles to km in the form 1:n. Check that your answer is close to the true ratio.

3. A prize of €10,000 is shared among 20 people. How much does each person get? How much would each person get if the prize were shared among 80 people?

4. A group of workers in an office do the Lotto each week. One week they won €400 and they received €16 each in prize money. If the prize had been €2,800, how much would each one have received? Solve this problem in two different ways.

5. A group of five tourists have sailed to a remote island, which has been cut off from the mainland due to stormy weather. They have enough food for 5 days if they eat 1kg per day. How many days will their food last if they eat (i) 200g per day (ii) 1.5kg per day?

6. The scale on a map is 1: 20,000. Find the distance in centimetres on the map representing an actual distance of 1km.

7. A room needs 500 tiles measuring 15cm x 15cm to cover the floor. How many tiles measuring 25cm x 25cm would be needed to tile the same floor?

8. Two students go on holidays planning to spend €40 per day for a holiday lasting 8 days. They end up spending €60 per day. How many days can the money last at this rate of spending?

9. Mark drives home in 2 hours if he drives at an average speed of 75km/h. If he drives at an average speed of 80km/h, how long will the journey take him?

10. The water supply for a community is stored in a water tower. A pumping station can fill the tower's tank at 600 litres per minute in 3 hours if no water is being drawn from the tank. How long will the tank take to fill if water is being drawn off at the rate of (i) 200 litres per minute (ii) 400 litres per minute?

Section O: Student Activity 11

Project Maths
Tionscadal Mata
Development Team

1. A chef can make 3 apple tarts in an hour. His helper can make 3 apple tarts in 2 hours. They need to make 27 apple tarts. How long will this take them working together?

2. Two taps are filling a bath. It takes one tap 4 minutes to fill the bath and the other tap 5 minutes. How long will it take to fill the bath with both taps filling at the given rates?

3. Olive has a report to type which is 10,000 words long. She can type at an average speed of 50 words per minute. How long will it take her to type the report typing at this speed? Her friend George can type at 60 words per minute. She gives George 3/5 of the report to type while she types the remainder. How long will it take now to type the report?

4. One combine harvester can harvest a field of corn in 4.5 hours. Another harvester can harvest the same field in 3 hours. If the farmer uses the two harvesters at the same time how long will it take to harvest the entire field?

5. A fruit grower estimates that his crop of strawberries should yield 70 baskets. His three children agree to pick the strawberries. On average, one child can pick 2 baskets in 1.5 hours, another child can pick 4 baskets in 2.5 hours and his third child can pick 5 baskets in 4 hours. How long will it take the 3 of them working together to pick 70 baskets of the fruit?

6. Write a question that involves combining 2 different rates.

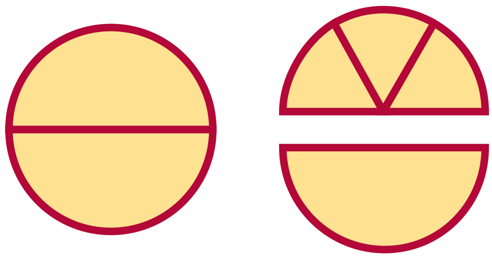
7. Write a question that involves combining 3 different rates.

 www.projectmaths.ie

Section J: Appendix 1: Partitioning

Dave's party:

Dividing 2 cakes among 3 people:

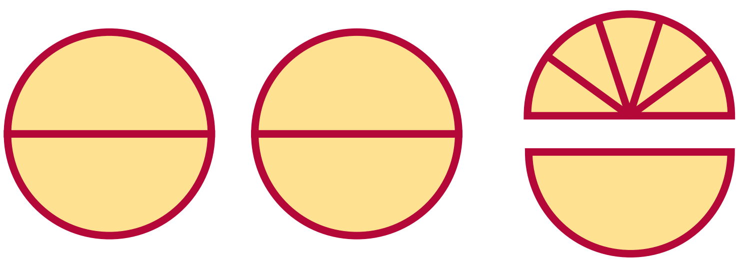


Each person gets $\frac{1}{2}+\frac{1}{6}$ **[**$\frac{1}{6}$ **is** $\frac{1}{3}$ **of** $\frac{1}{2}$**]**

$$\frac{1}{2}+\frac{1}{6}=\frac{3}{6}+\frac{1}{6}=\frac{4}{6}=\frac{2}{3}$$

Ray's party:

Dividing 3 cakes among 5 people



Each person gets $\frac{1}{2}+\frac{1}{10}$ **[**$\frac{1}{10}$ **is** $\frac{1}{5}$ **of** $\frac{1}{2}$**]**

$$\frac{1}{2}+\frac{1}{10}=\frac{3}{5}$$