Number & Algebra: Strands 3 & 4

#1
A Relations Approach to Algebra: Linear Functions
#2
A Relations Approach to Algebra: Quadratic, Cubic & Exponential Functions
#3
Applications of Sequences & Series
#4
Applications of Sequences & Series

Project Maths Tionscadal Mata
Development Team

3

Name: _____

School: _____

Linking Depreciation and Compounding to Prior Knowledge on Exponential Functions using Tables, Graphs and Formulae

2007 JC HL Q1 (b)

A snowman has a mass of 12 kg.

It melts at a rate of 0·2% of its mass per minute.

What will be the mass of the

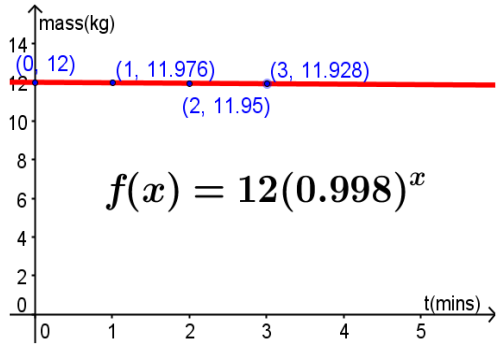
snowman after 3 minutes?

Give your answer correct to 2 decimal places.

Poorly answered.

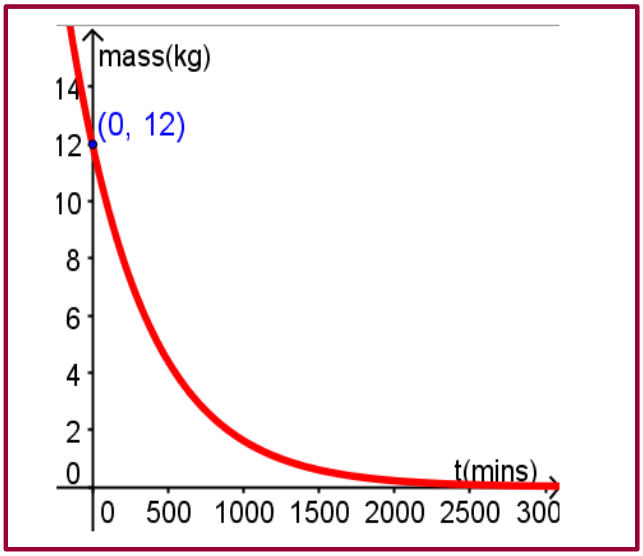
Common errors: ***1.*** *Ignoring cumulative loss of mass.*

2. *Mistake in % or decimal.*



Looks linear.....???

The bigger picture.....



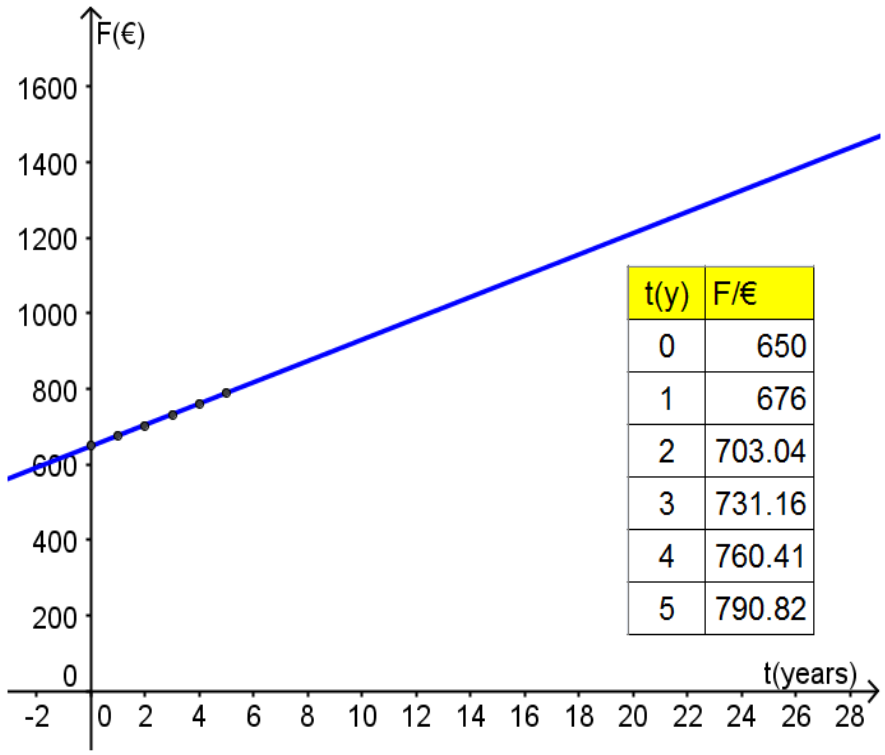
€650 is deposited in a fixed interest rate bank account. The amount in the account at the end of each year is shown in the following table.

End of year	1	2	3	4	5
Final value/€	676	703.04	€731.16	760.41	790.82

(a) Explain whether or not the relationship between final value and time can be modelled by a linear, quadratic or exponential function or by none of these?

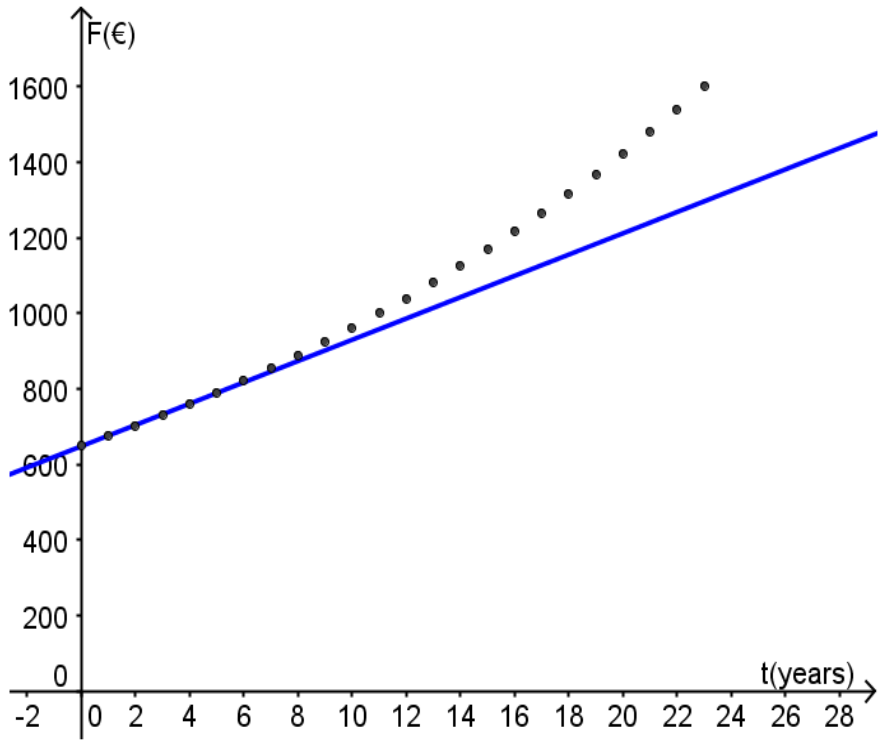
(b) If you plot a graph of final value against time what does the graph look like for this limited range of times?

Looks linear.....



t(y)	F/€
0	650
1	676
2	703.04
3	731.16
4	760.41
5	790.82

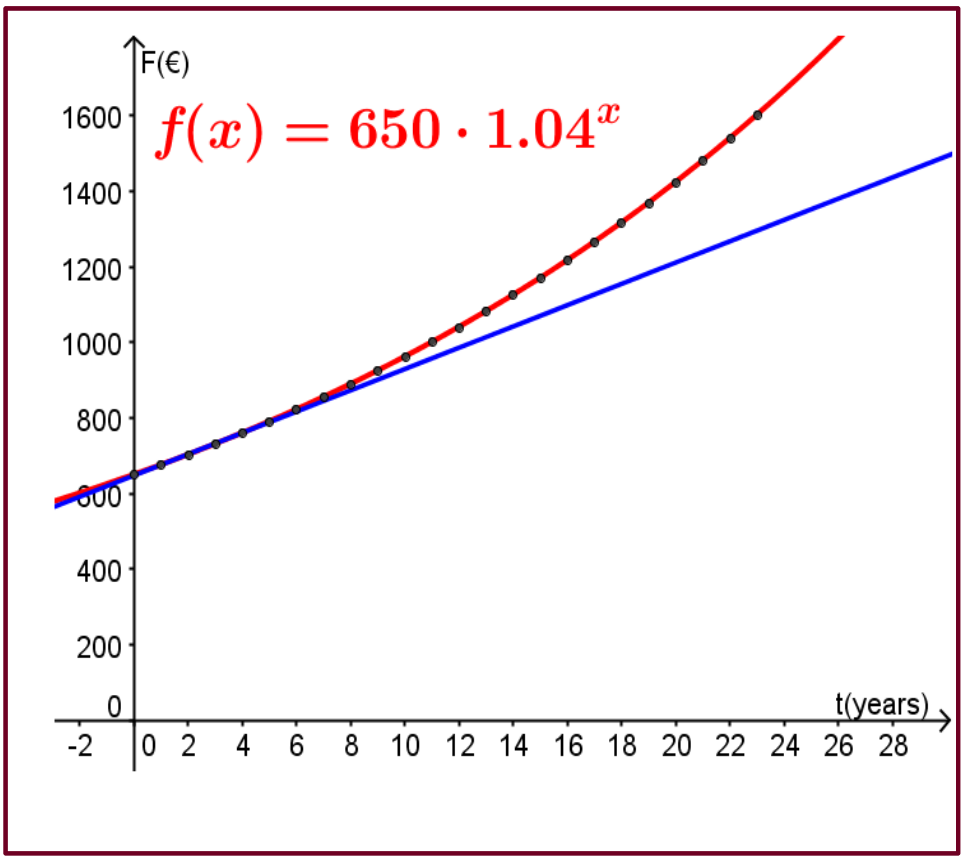
On further investigation.....



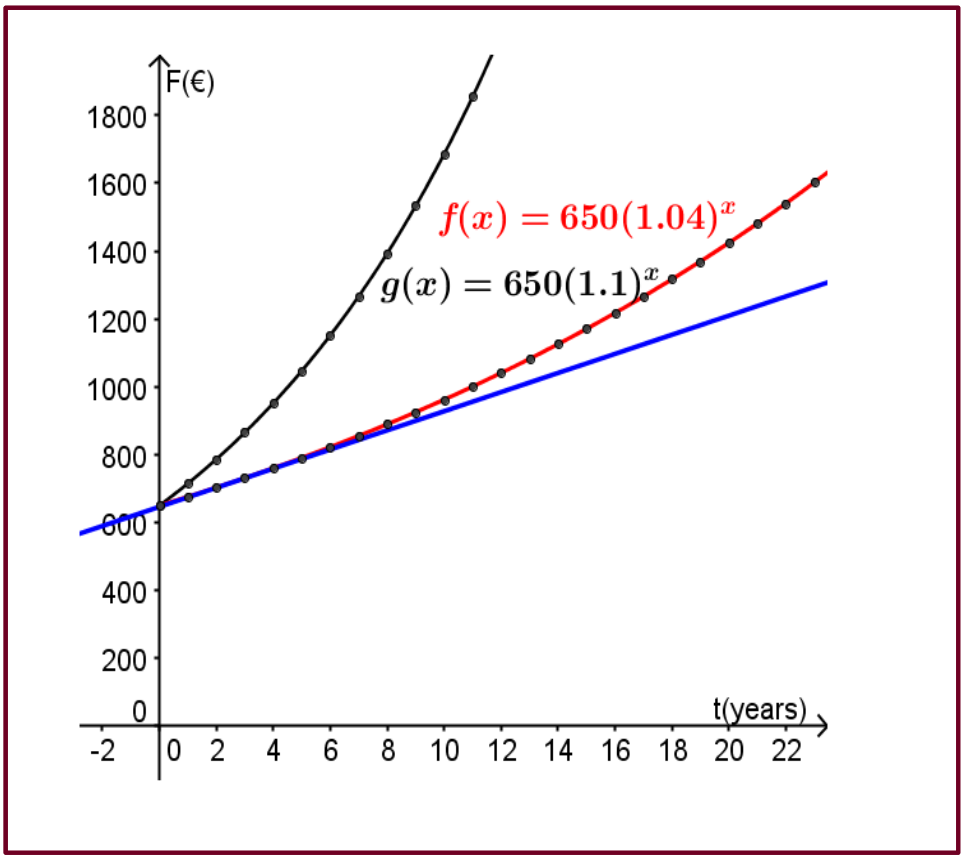
t(y)	F/€
0	650
1	676
2	703.04
3	731.16
4	760.41
5	790.82
6	822.46
7	855.36
8	889.57
9	925.15
10	962.16
11	1000.65
12	1040.67
13	1082.3
14	1125.59
15	1170.61
16	1217.44
17	1266.14
18	1316.78
19	1369.45
20	1424.23
21	1481.2
22	1540.45
23	1602.07

What formula expresses the final value in *t* years given an initial value of €650?

Final value is growing exponentially



Comparing a 10% interest rate to a 4% interest rate



Applying Rules for Indices

Complete the following table, without the use of a calculator.
Leave your answers in index form:

Before multiplying	After multiplying	Before multiplying	After multiplying
1. $a^7 \times a^3$		6. $1.02^3 \times 1.02^3$	
2. $8^3 \times 8^2$		7. $1.14^5 \times 1.14^2$	
3. $8.2^5 \times 8.2^2$		8. $1.06^4 \times 1.06$	
4. $(6.4)^5 (6.4)^2$		9. $(1.08)^5 (1.08)$	
5. $1.3^2 \times 1.3^5$		10. 1.07×1.07^5	

Evaluate Using the Calculator *(prior knowledge for method 2)*

1. 7^4

2. 4.5^4

3. 1.8^5

4. 1.06^6

5. 1.325^5

6. $\left(\frac{3}{2}\right)^7$

7. $\left(\frac{1}{2}\right)^4$

8. $10(3)^4$

9. $100(6)^3$

10. $1000(2.5)^3$

11. $300(1.03)^6$

12. $2000(1.025)^5$

13. $250(1.16)^4$

14. $400(1.08)^4$

Final Value Using Compounding (two methods)

Method 1	
Value of the gift (P)	€5000
Interest for the 1st year (I_1) (4% of €5 000)	€200
F_1 =Final value (end of year 1)	€5200
Interest for the 2nd year (I_2)	€208
F_2 = Final Value (end of year 2)	€5408
Interest for the 3rd year (I_3)	€216.32
F_3 =Final Value (end of year 3)	€5624.32

$0.04 = i$

$1.04 = 1 + i$

Write an expression for the final value F in terms of P, i and t.

Method 2

Value at end of year 1 €5000 × **1.04** = €5200	=	Value at end of year 1 €5000 × **1.04**
Value at end of year 2 €5200 × **1.04** = €5408	=	Value at end of year 2 €5000 × **1.04^2** Check with a calculator
Value at end of year 3 €5408 × **1.04** = €5624.32	=	Value at end of year 2 €5000 × **1.04^3** Check with a calculator

What % of P is F_1?	104%	Express this as a number and use it to calculate F_1	1.04
What % of F_1 is F_2?	104%	Express this as a number and use it to calculate F_2	1.04
What % of F_2 is F_3 ?	104%	Express this as a number and use it to calculate F_3	1.04

Finding Roots

Revision using the Calculator

$2 \times 2 \times 2 \times 2 \times 2 = 32$, therefore the 5th root of 32 is 2.

Verify this using the root key $\left[\sqrt[b]{a}\right]$

Evalaute the following:

(a) $625^{\frac{1}{4}}$

(b) $\sqrt[12]{1.043}$

(c) $1.043^{\frac{1}{365}}$

(d) $\sqrt[10]{\frac{5000}{3750}}$

Finding Number of Years (or other time periods) LCHL

Fiona has put €5000 into a savings account at 7% AER.
She needs €10 000 in order to build an extension to her house.
How many years will it take for Fiona to reach her target of €10 000 ?
Give your answer correct to one decimal place.

P	*i*	(1+*i*)	*t*	*F*
€5000	0.07	1.07		€10000

N.B. Prior knowledge of logarithms required

$10000 = 5000(1.07)^t$

$2 = (1.07)^t$

$\log_{1.07} 2 = t$

$t = 10.24$ years

Reducing Balance

Jillian and Noel are each going to buy a games console. It costs €500 and they are getting a loan from the credit union . Jillian says "I am making a lot of money at the moment so I can afford to pay €100 per month." Noel says that he can only afford €80 per month. The credit union is charging them a monthly interest rate of 1% to be paid at the end of each month.
Find their outstanding balances at the end of each month.

(a) A loan is taken out

(b) After 1 month interest is added on

(c) The person then makes his/her monthly repayment.
This process is then repeated until the loan is fully paid off.

Reducing Balance

Compare the total interest paid by Jillian and Noel.

Total interest J = €15.55
Total interest N = €18.98

Compare the time taken by Jillian and Noel to pay off the loan.

t_J = 6 months
t_N = 7 months

Jillian

Initial loan	€500.00
Interest 1	€5.00
Total	€505.00
Payment 1	€100.00
Balance 1	€405.00
Interest 2	4.05
Total	€409.05
Payment 2	€100.00
Balance 2	€309.05
Interest 3	3.0905
Total	€312.14
Payment 3	€100.00
Balance 3	€212.14
Interest 4	2.121405
Total	€214.26
Payment 4	€100.00
Balance 4	€114.26
Interest 5	1.14261905
Total	€115.40
Payment 5	€100.00
Balance 5	€15.40
Interest 6	0.154045241
Total	€15.56
Payment 6	€15.56
Balance 6	€0.00

Noel

Initial loan	€500.00
Interest 1	€5.00
Total	€505.00
Payment 1	€80.00
Balance 1	€425.00
Interest 2	4.25
Total	€429.25
Payment 2	€80.00
Balance 2	€349.25
Interest 3	3.4925
Total	€352.74
Payment 3	€80.00
Balance 3	€272.74
Interest 4	2.727425
Total	€275.47
Payment 4	€80.00
Balance 4	€195.47
Interest 5	1.95469925
Total	€197.42
Payment 5	€80.00
Balance 5	€117.42
Interest 6	1.174246243
Total	€118.60
Payment 6	€80.00
Balance 6	€38.60
Interest 7	0.385988705
Total	€38.98
Payment 7	€38.98
Balance 7	€0.00

Depreciation

A company buys a new lorry for €50 000. After 4 years it needs to sell the lorry. The value of the lorry reduces by 15% each year. What is the value of the lorry after 4 years?

Depreciation

Method 1	
Original Value of Lorry P	**€50000**
Depreciation in year 1 =€50000(0.15)	**€7500**
F_1 =Final value (end of year 1)	**€42500**
Depreciation in year 2	**€6375**
F_2 = Final Value (end of year 2)	**€36125**
Depreciation in year 3	**€5418.75**
F_3 =Final Value (end of year 3)	**€30706.25**

Method 2

Value at end of year 1 €50000 × **0.85** = €42500	=	Value at end of year 1 €50000 × **0.85**
Value at end of year 2 €42500 × **0.85** = €36125	=	Value at end of year 2 €50000 × **0.85^2** **Check with a calculator**
Value at end of year 3 €536125 × **0.85** = €30706.25	=	Value at end of year 3 €50000 × **0.85^3** **Check with a calculator**

$0.04 = i$

$1.04 = 1 + i$

Write an expression for the final value F in terms of P, i and t.

What % of P is F_1?	85%	Express this as a number and use it to calculate F_1	0.85
What % of F_1 is F_2?	85%	Express this as a number and use it to calculate F_2	0.85
What % of F_2 is F_3 ?	85%	Express this as a number and use it to calculate F_3	0.85

AER, EAR, CAR
&
Interest Rates other than Annual

Savings and Investments

AER (annual equivalent/effective rate) tells you what interest you will earn annually, which depends on how often interest is added.

- ✓ Used for savings and investments
- ✓ It may or may not include charges;
- ✓ Allows investors to make comparisons between savings accounts which pay interest at different intervals
- ✓ Takes into consideration the effect of compounding interest

The financial regulator's office considers the terms AER/EAR and CAR all to be equivalent. The term CAR is approved for use in relation to tracker bonds – for other investment products the regulator considers the acronym AER or EAR should be used.

AER

Leaving Certificate 2010 Sample Paper 1 Foundation Level Q2

A sum of €5000 is invested in an eight-year government bond with an annual equivalent rate **(AER)** of 6%.
Find the value of the investment when it matures in eight years' time.

$$F = P(1+0.06)^8 = €7969.24$$

Leaving Certificate 2010 Sample Paper 1 Ordinary Level Q2

(a) A sum of €5000 is invested in an eight-year government bond with an annual equivalent rate **(AER)** of 6%. Find the value of the investment when it matures in eight years' time.
(b) A different investment bond gives 20% interest after 8 years. Calculate the AER for this bond.

$$6000 = 5000(1+i)^8 \Rightarrow (1+i) = 1.2^{\frac{1}{8}} = 1.02305 \Rightarrow i = 0.02305$$

$$AER = 2.305\%$$

Unwrap the Terminology

Advertisement Christmas 2010 with AER

15 month fixed term savings account

We Really do Want your Money

2011

15 month fixed term savings account

Justify that that 5.25% fixed for 15 months is the same as 4.18% AER