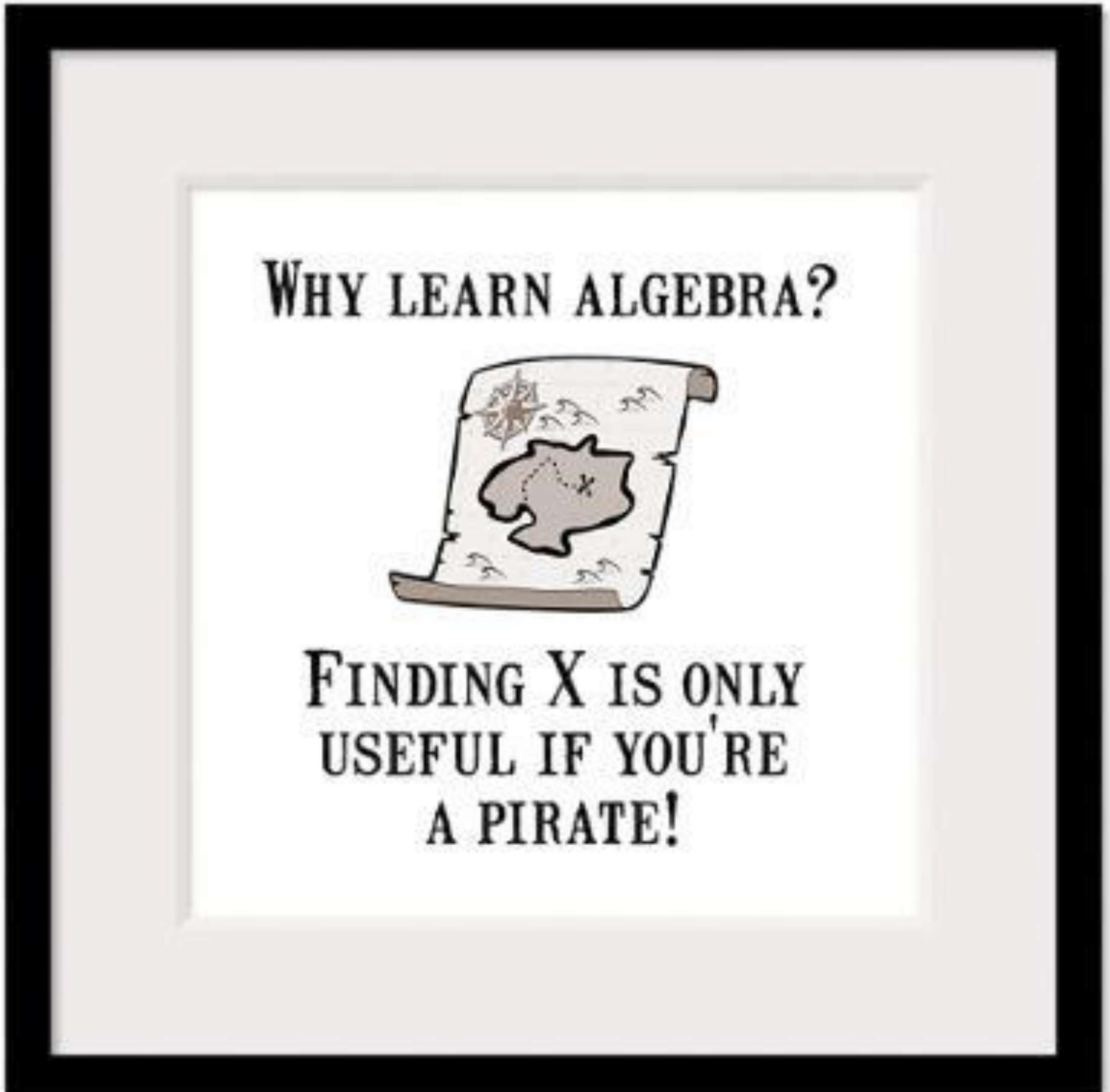

WHY LEARN ALGEBRA?
X
FINDING X IS ONLY
USEFUL IF YOU'RE
A PIRATE!

Problem solving

"Build a better mouse trap and the world will beat a path to your door"

Ralph Waldo Emerson

Invented in 1894 by William C Hooker

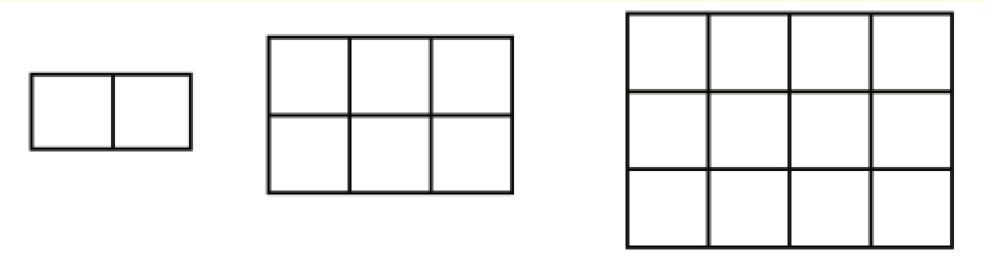
Global competitions each year to improve on the basic design

Problem solving

But.. mice have started to use problem solving strategies

Activity 8 [Growing Rectangles]

Complete the next two patterns in this sequence of rectangles.



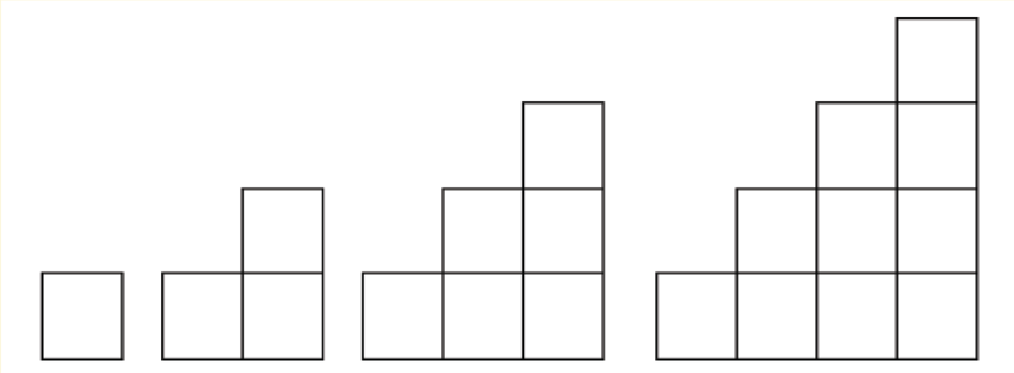
Look at the pattern of growing rectangles. Make a table for the number of tiles in each rectangle for rectangles of heights from 1 to 10. Make observations about the values in the table.

Activity 9 [Staircase Towers]

Example 3: Staircase Towers JC HL

Look at the staircases below. Make a table representing the relationship between the total number of tiles and the number of towers. Make observations about the values in the table. What would a graph look like? Would it be linear? How do you know? Make a graph to check your prediction.

Allow students to continue the pattern themselves and make their own observations. The following questions could be used as prompt questions if necessary.



Exploring Numbers

"...it is crucial that algebraic thinking is developed in parallel with arithmetic thinking from an early age.

Not only does this result in a deeper understanding of our number system but also forms a strong basis on which to build formal algebraic thinking in the later years."

Mulligan, Cavanagh & Keanan-Brown

Activity 10 [Perfect Squares]

The table below gives the first thirty two non-negative integers arranged in rows of four.

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

Today's Question

Prove that a perfect square is always a multiple of four or one more than a multiple of four.

0	1	2	3
4	5	6	7
8	9	10	11
12	13	14	15
16	17	18	19
20	21	22	23
24	25	26	27
28	29	30	31

*

Solutions

$$(4n)^2 = 16n^2$$
$$= 4(4n^2)$$
$$= 4K$$

$$(4n+2)^2 = 16n^2 + 16n + 4$$
$$= 4(4n^2 + 4n + 1)$$
$$= 4K$$

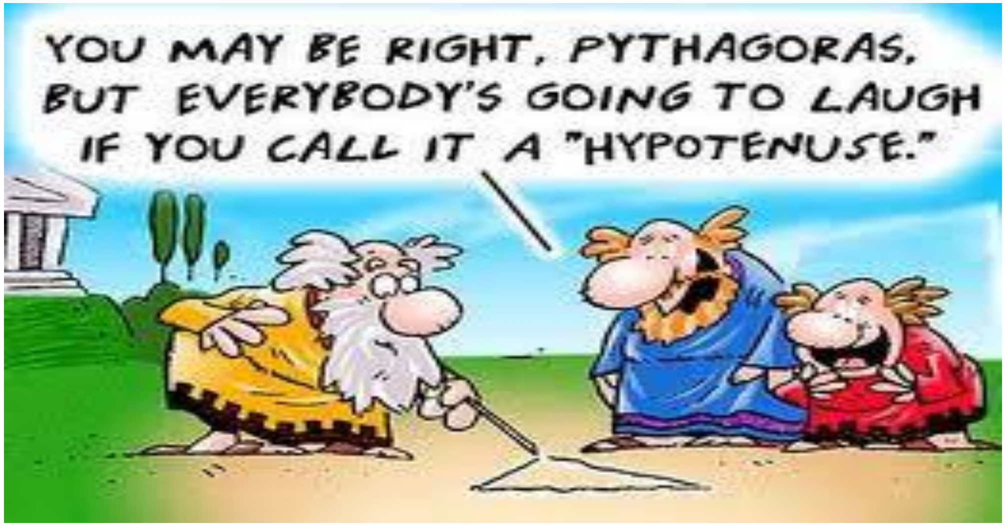
$$(4n+1)^2 = 16n^2 + 8n + 1$$
$$= 4(4n^2 + 2n) + 1$$
$$= 4K + 1$$

$$(4n+3)^2 = 16n^2 + 24n + 9$$
$$= 4(4n^2 + 6n + 2) + 1$$
$$= 4K + 1$$

Extension Question

Show that the sum of two squares is never three more than a multiple of four.

Activity 6 [Pythagorean Triples]



Prove that a Pythagorean triple can't consist of three odd numbers.

Solution

Pythagorean triples take the form: $a^2 = b^2 + c^2$

Odd numbers take the form: $2n + 1$

If a, b and c are odd....

$(2n+1)^2 = (2k+1)^2 + (2m+1)^2$

Solution

$$4n^2 + 4n + 1 = 4(n^2 + n) + 1 \ldots$$

$$4k^2 + 4k + 1 = 4(k^2 + k) + 1 \ldots$$

$$4m^2 + 4m + 1 = 4(m^2 + m) + 1$$

$$(odd)^2 = (odd)^2 + (odd)^2 \quad ?$$

False

JCHL Sample paper (1) 2013 (phase3)

Given any two positive integers m and n ($n>m$), it is possible to form three numbers a, b and c where:

$$a = n^2 - m^2, \qquad b = 2nm, \qquad c = n^2 + m^2$$

These three numbers a, b and c are then known as a "Pythagorean triple".

(a) For $m = 3$ and $n = 5$ calculate a, b and c.

(b) If the values of a, b, and c from part **(a)** are the lengths of the sides of a triangle, show that the triangle is right-angled.

(c) If $n^2 - m^2$, $2nm$ and $n^2 + m^2$ are the lengths of the sides of a triangle, show that the triangle is right angled.

JCHL Sample paper (1) 2013 (phase3) Solution

Euclid's formula states that the integers form Pythagorean triples.

As $a^2+b^2=c^2$

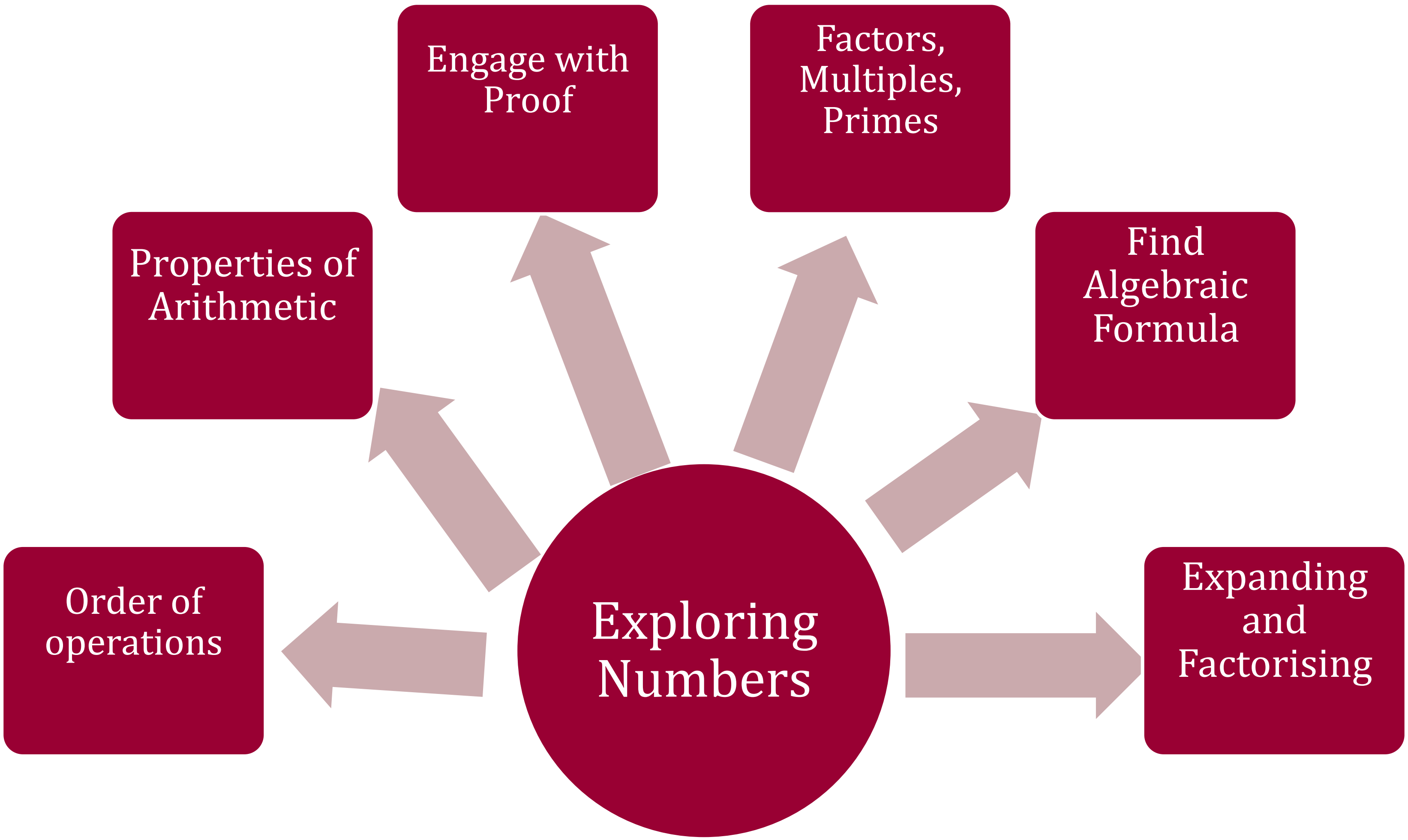
Then, $(n^2-m^2)+(2nm)^2=(n^2+m^2)^2$

$N^4-2n^2m^2 + m^4 + 4m^2n^2 = n^4 + 2m^2n^2 + m^4 =(n^2+m^2)^2$.

Pythagorean triples must have 3 positive integers. 1,1, root2 are not a pythagorean triple.

It is based on the difference of squares of any two consecutive numbers is always an odd number!!
There are an infinite number of odd squares.....

Exploring Numbers



Why Choose This Problem?

- Interesting.
- Many approaches.
- Accessible.
- Layers.
- Linked to syllabus.
- Lends itself to group work and promotes discussion.
- Requires students to explain their thinking.

Syllabus

Synthesis and problem-solving skills

– explore patterns and formulate conjectures

– explain findings

– justify conclusions

– communicate mathematics verbally and in written form

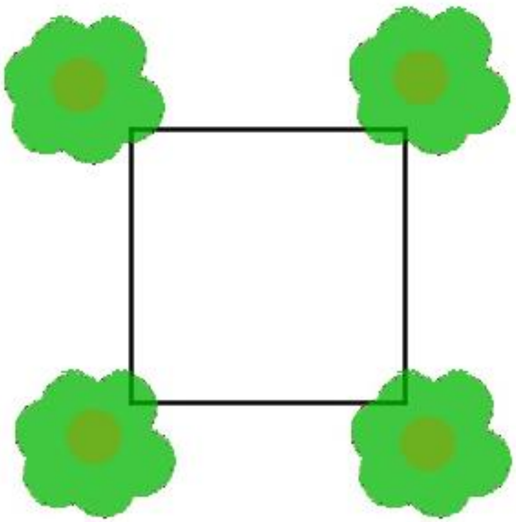
– apply their knowledge and skills to solve problems in familiar and unfamiliar contexts

– analyse information presented verbally and translate it into mathematical form

– devise, select and use appropriate mathematical models, formulae or techniques to process information and to draw relevant conclusions.

Activity 11 [Car Park Space]

A client wants to double the area of his car park. He insists on retaining a square shape and as many of the trees as possible.



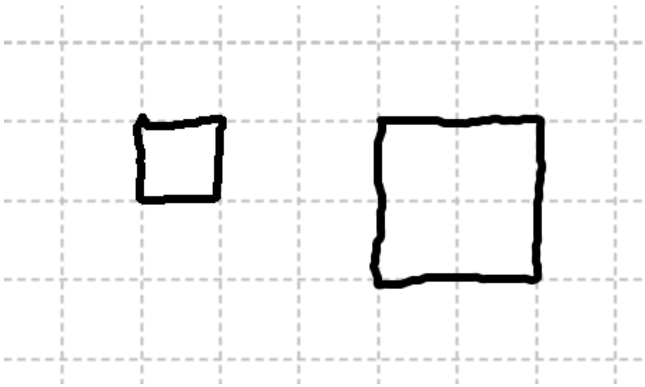
What is the best solution?

Construction

Carpark

Some Approaches

Double sides and see what happens



Improving an estimate

$1.4^2 = 1.96$

$1.45^2 = 2.1025$

$1.42^2 = 2.0164$

$1.41^2 = 1.9881$

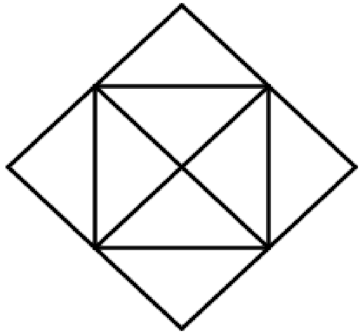
$1.415^2 = 2.002225$

Algebraic Symbols

$$x^2 = 2$$

$$x = \sqrt{2}$$

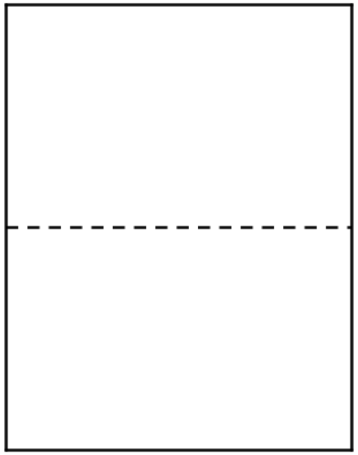
Folding (141%)



Activity 14 [Photocopying Paper]

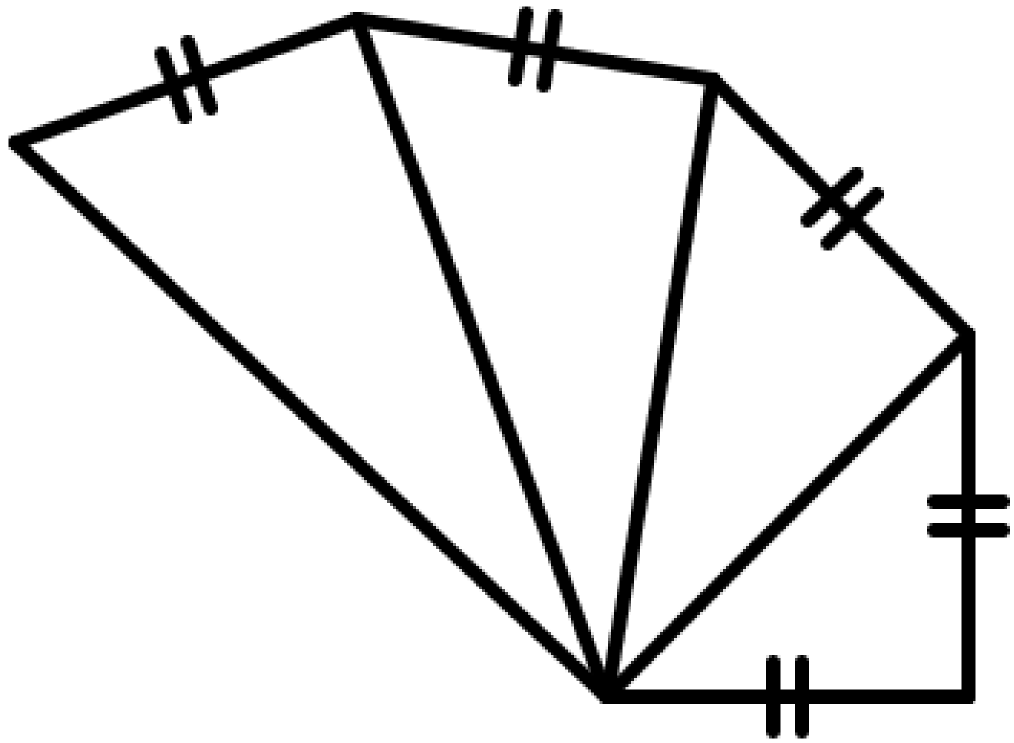
A paper manufacturer wants to design a rectangular piece of paper with a special property.

He wants the ratio of the long side to the short side to be such that if he was to fold the page in half the 'new' long side and the 'new' short side will be in the same ratio as the original.



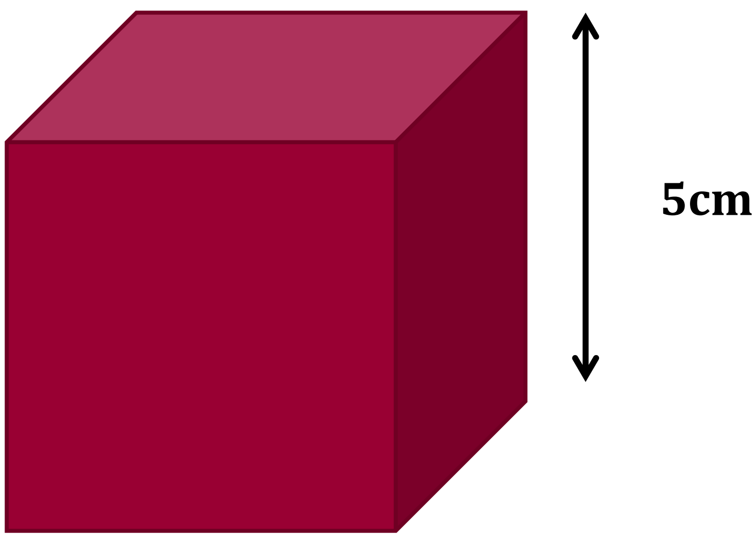
Extend to..

- How would you make the car park three times the size of the original?(retaining the square shape)



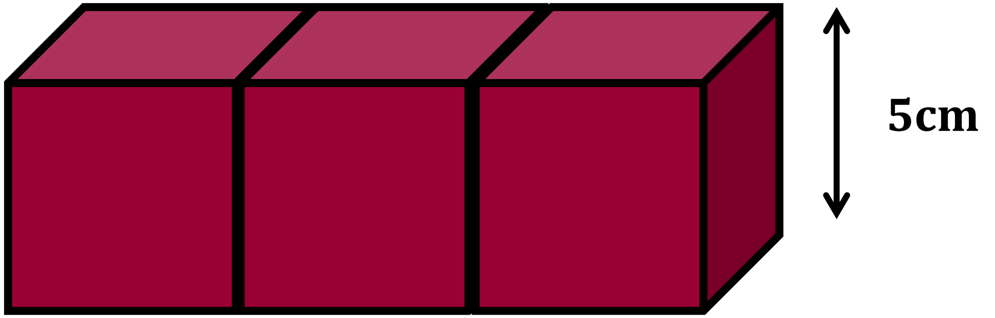
Activity 13 [Nets]

- What is the surface area of a cube with edge length of 5 cm?



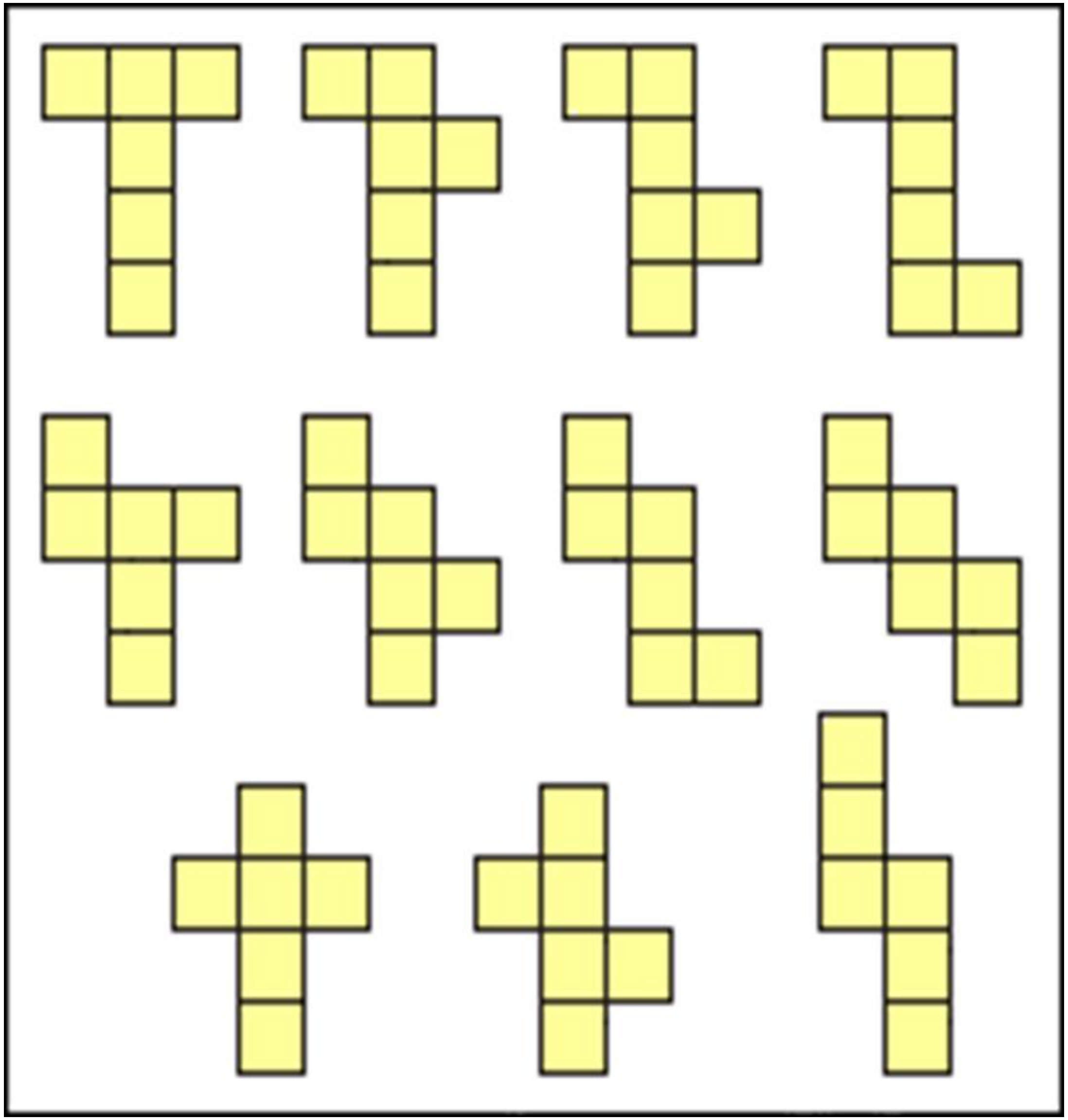
Activity 13 [Nets]

- Three such cubes are joined together side by side and wrapped with paper (no excess paper is used, i.e no overlap).
- What is the area of the paper needed?
- Draw a net for the shape of one cube.



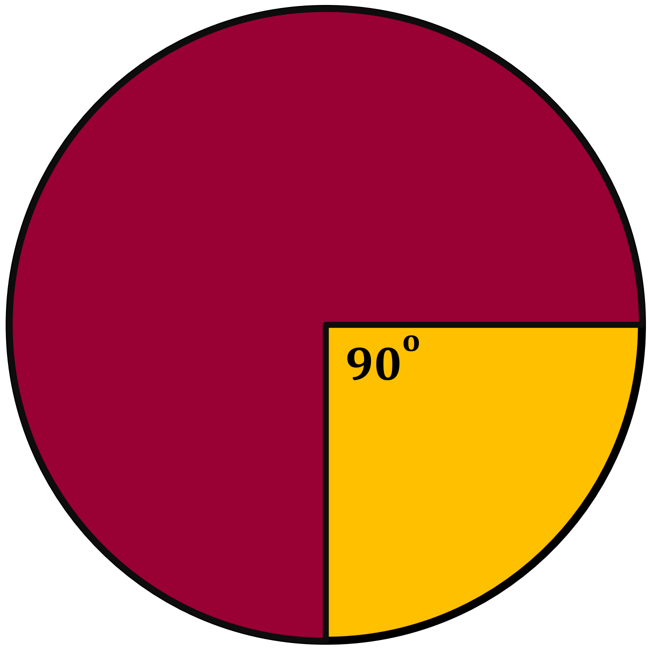
Possible Solutions

Nets of a cube

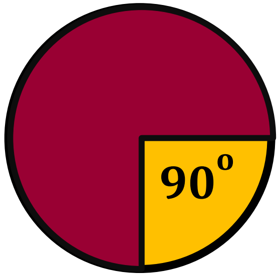


Activity 13 [Nets]

- The shaded region is cut from a circular piece of plastic of radius 10cm.
- What shape is this the net of?
- How much liquid would this hold?



Solution Cone Net



Link: Net of a cone

Area of Circle $= \pi r^2 = 100\pi \text{ cm}^2$

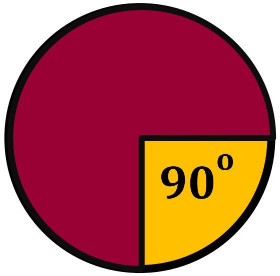
Area of ¾ of Circle $= 75\pi$

Surface Area of Cone $= \pi r l = 75\pi$

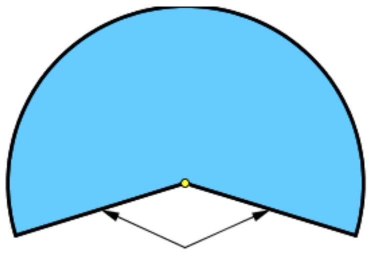
$75 = rl$ (we know $l = 10$ cm)

Thus: $\frac{75}{10} = r = 7.5\ cm$

Solution Cone Net



Link: Net of a cone



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$75 = rl$ (we know $l = 10$ cm)

Thus: $\frac{75}{10} = r = 7.5\ cm$

Finding h, use Pythagoras

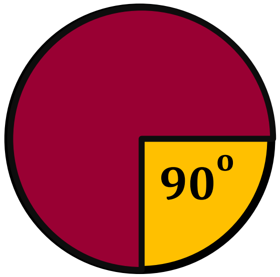
$10^2 = 7.5^2 + h^2$

$100 - 56.25 = h^2$

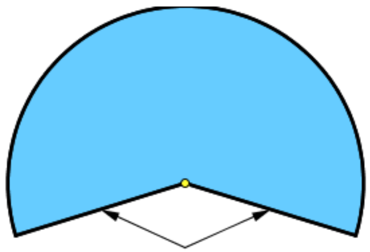
$43.75 = h^2$

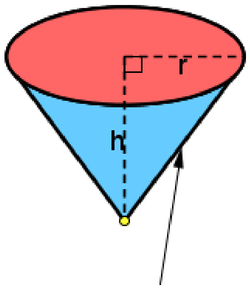
$6.6 \text{ cm} = h$

Solution Cone Net



Link: Net of a cone





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$100 - 56.25 = h^2$

$43.75 = h^2$

$6.6 \text{ cm} = h$

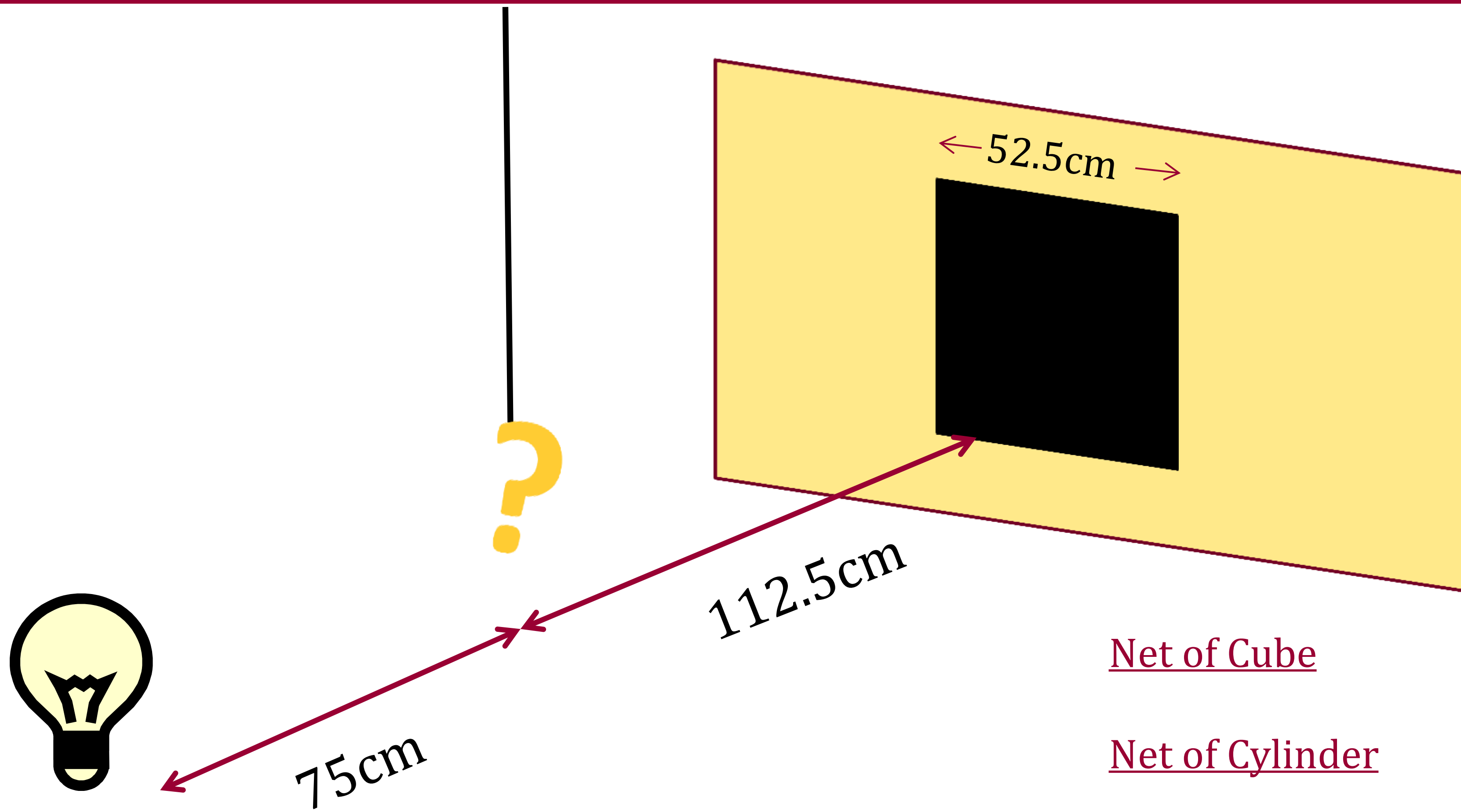
Volume of a cone

$= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 7.5^2 \times 6.6$

$= 388.77 \cong 389 \text{ cm}^3$

What am I?



Net of Cube

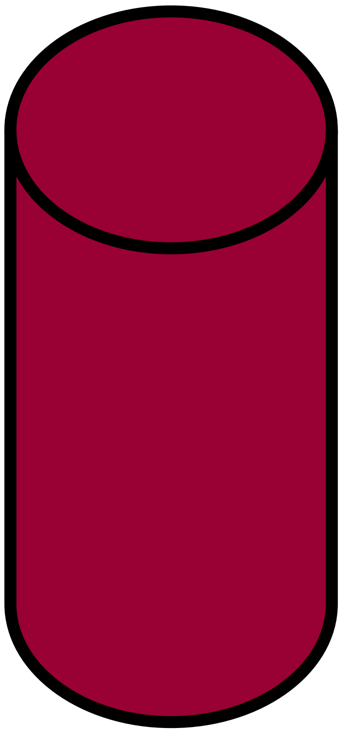
Net of Cylinder

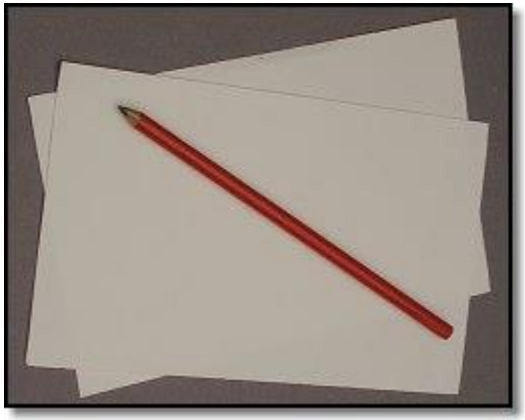
Net of Prism

What is the length of the square side of the object?

Activity 14 [Design a Pencil Case]

- Design a cylindrical pencil case using fabric the same size as A4 paper. Try to make the pencil case as large as possible.





Points to ponder

Extension

Activity 14 [Design a Pencil Case]

Getting started

The width of the A4 sheet could determine the circumference of the base of the cylinder. Then the diameter of the base restricts how high the cylinder can be - can you work out the dimensions of this cylinder?

The length of the A4 sheet could determine the circumference of the base of the cylinder.

The width of the A4 sheet could determine the height of the cylinder.

The length of the A4 sheet could determine the height of the cylinder.

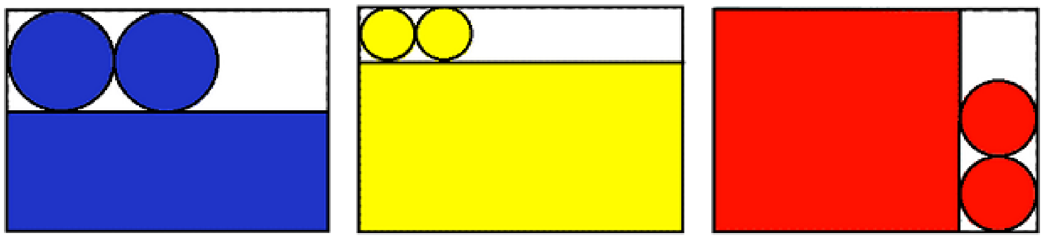
Activity 14 [Design a Pencil Case]

Efficient Cutting

Specialists in rich mathematics

A cylindrical container can be made by using two circles for the ends and a rectangle which wraps round to form the body.

To make cylinders of varying sizes, the three pieces can be cut from a single rectangular sheet in several ways. For example:

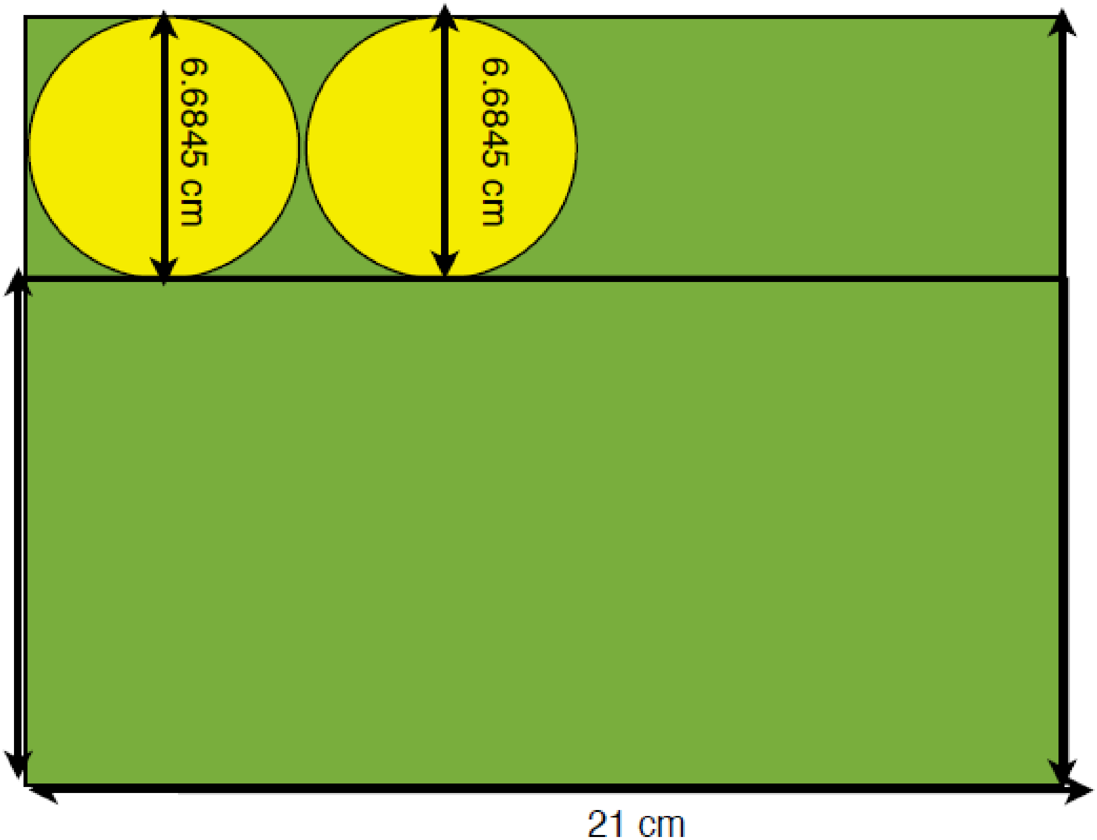


Using a single sheet of A4 paper, make the cylinder with the largest volume. The cylinder must be closed off by a circle at each end.

What are its dimensions?

Activity 14 [Design a Pencil Case]

1st idea's Diagram



1st idea

One of the side equals the circle's circumference so it'll not waste any space. I chose the side that's 21 cm as the circle's circumference.

1st idea's Method

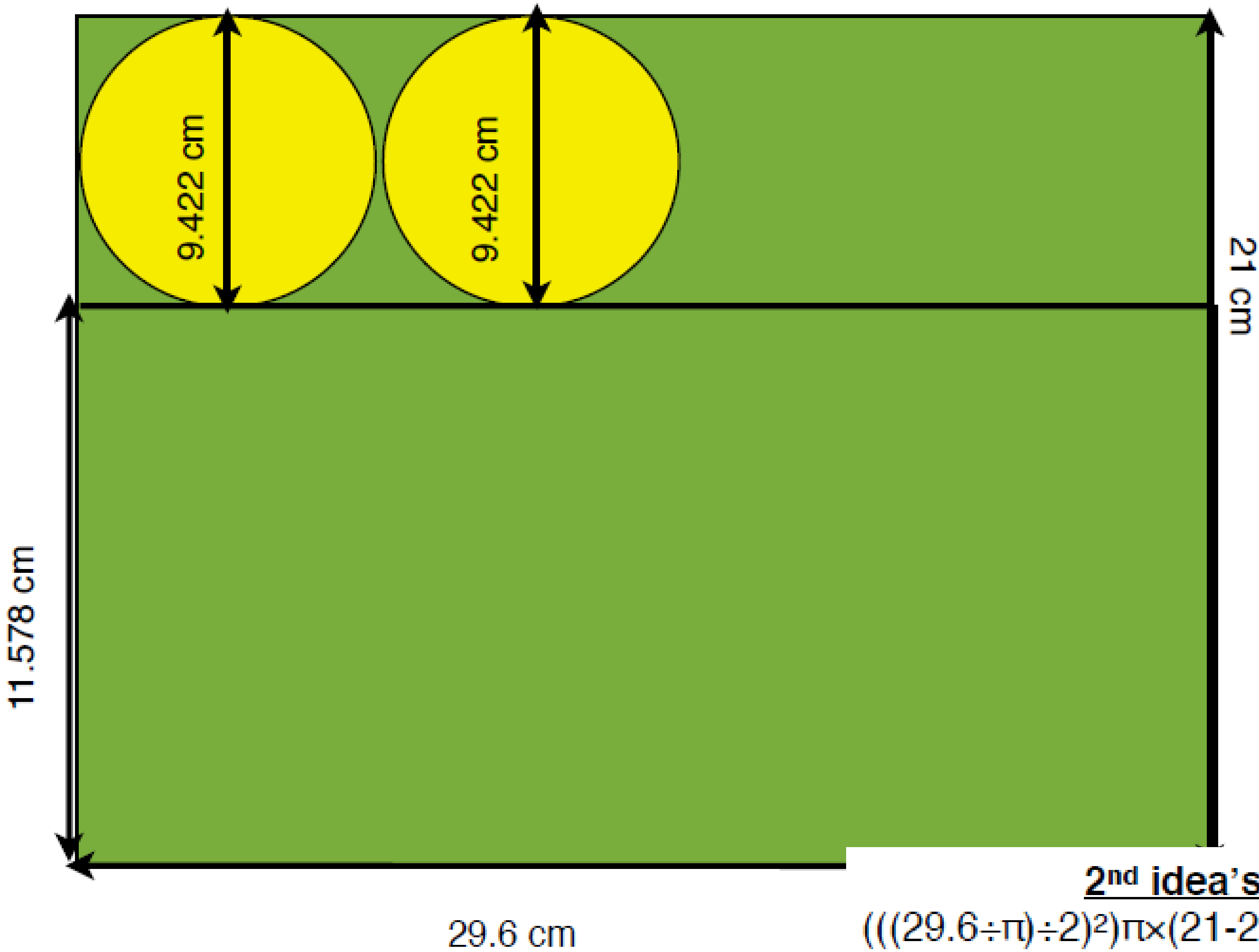
$(((21\div\pi)\div2)^2)\pi\times(29.6-21\div\pi)=804.1886121$

$21\div\pi\approx6.6845$ $6.6845\div2=3.34225$ $3.4225^{23}\approx11.1707$

$11.1707\times\pi\approx35.09366$ $29.6-6.6845=22.9115$

$35.09366\times22.9115\approx$ **804.0485**

Activity 14 [Design a Pencil Case]



2nd idea

One of the side equals the circle's circumference so it'll not waste any space. I chose the side that's 29.6 cm as the circle's circumference. It was bigger because the circle's area got bigger.

2nd idea's Method

$(((29.6 \div \pi) \div 2)^2)\pi \times (21 - 29.6 \div \pi) = 807.2529233$

$29.6 \div \pi \approx 9.422 \quad 9.422 \div 2 = 4.711$

$4.711^2 \approx 22.1935 \quad 22.1935 \times \pi \approx 69.723$

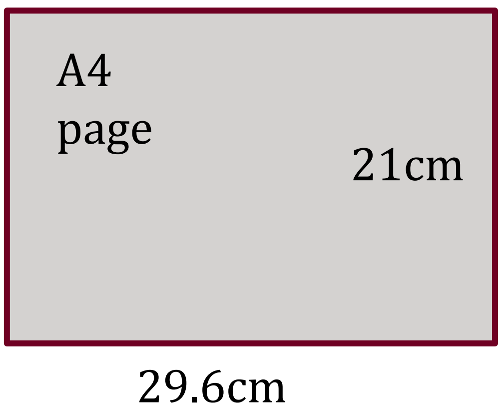
$21 - 9.422 = 11.578$

$69.723 \times 11.578 \approx \mathbf{807.2529}$

Activity 14 [Design a Pencil Case]

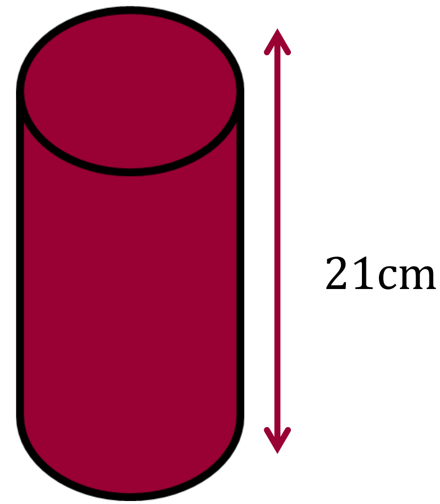
Let the width of the paper fix the height of the cylinder, then h=21
If r is the radius, then the length of the rectangle is $2\pi r$ so if we fit the rectangle plus the circle along the 29.6cm length we have

$$2r + 2\pi r = 29.6$$



$$r(2 + 2\pi) = 29.6$$

$$r = \frac{29.6}{2 + 2\pi}$$

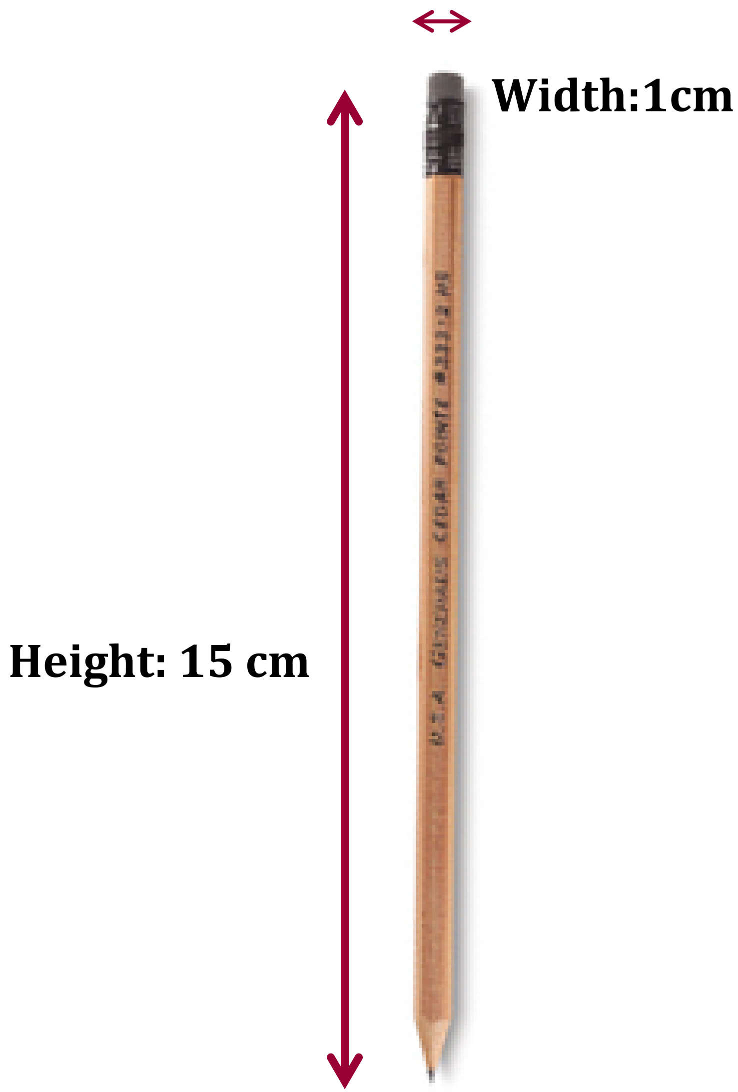


So the volume of the cylinder is:

$$\pi r^2 h = \pi \times \frac{29.6^2}{(2 + 2\pi)^2} \times 21 \approx 842$$

Design a pencil case

- **Extension Question:**
- Here is a picture of an average pencil.
  What is the maximum number of pencils which will fit in the pencil case you have designed.



Catering for Learning diversity

Half the students have finished an exercise when the other half have only just begun.

The stronger students get bored if I spend time explaining to the weaker ones.

We've got a syllabus to get through but most of the students are already behind.

The stronger students dominate.

The weaker students sit at the back and start disrupting the lesson.

The weaker students don't even try.

I don't know where to pitch my lesson.

The weaker students are always asking me things in their own language and want everything explained in it.

Some of the weaker students try so hard but they still get bad marks.

Problem Solving Strategies

Understan

•Is the prob

•Do you unc

•What are y

•Can you re:

•Can you thi
understand

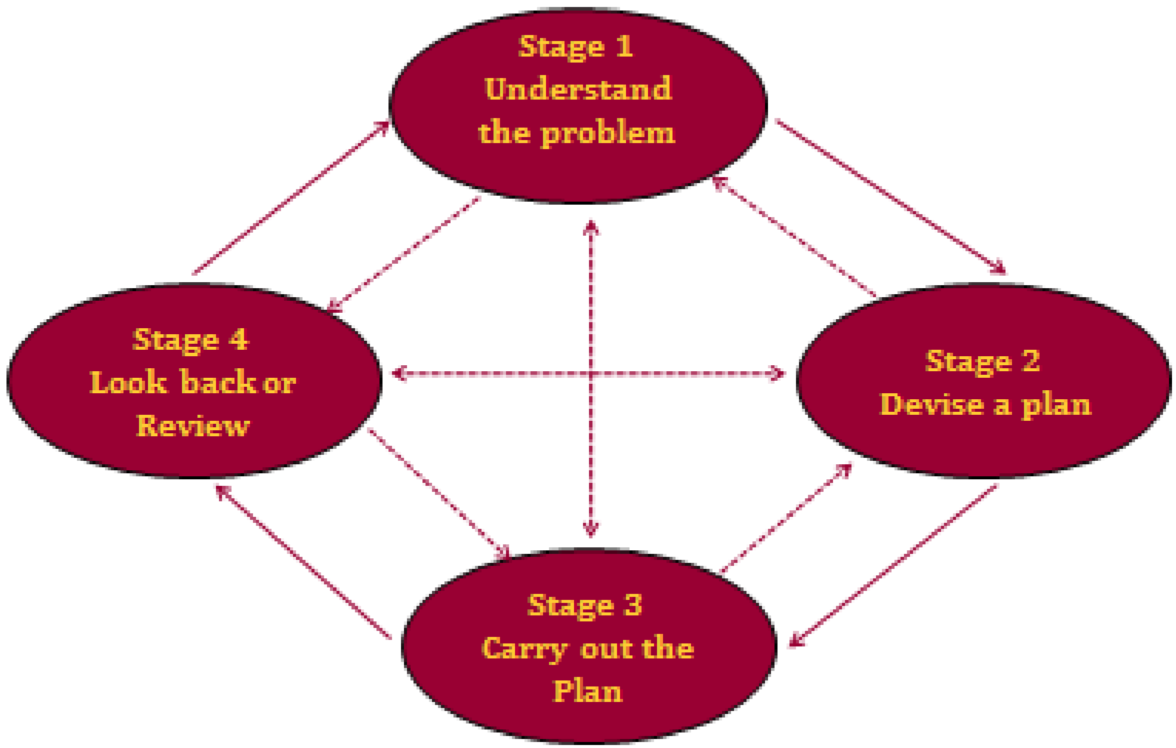
•Is there en

•Do you nee

Review your Plan

e done

Four Stage Problem Solving Process



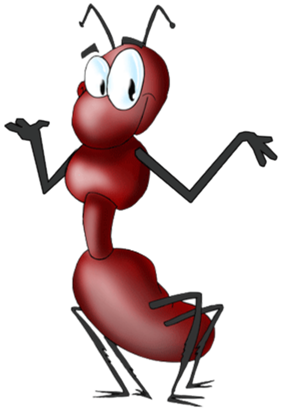
(Pólya, 1945)

Understanding the problem

- Is the problem well defined?
- Do you understand all the words used in stating the problem?
- What are you asked to find or show?
- Can you restate the problem in your own words?
- Can you think of a picture or a diagram that might help you understand the problem?
- Is there enough information to enable you to find a solution?
- Do you need to ask a question to get the answer?

Activity A1 [Exploration]

- An ant is crawling in a straight line from one corner of a table to the opposite corner. He bumps into a cube of sugar. He decides to climb over it and then continues along his intended route. How much did the detour add to his journey?



Activity A2 [Exploration]

Mrs. Smith has two twin girls. While on a day out, they find a bubble gum machine. The machine sells red bubble-gum and blue bubble gum. Each girl wants a piece of gum, but as they are twins, they must have the SAME colour gum.

If each turn on the bubble-gum machine costs 10 cents for one piece of gum, what is the maximum amount of money Mrs Smith must spend in order to ensure both girls have the same colour gum?

Hint

Using a table, we can view the outcomes like this;

Turn number	Gum Colour	Running cost (c)
1	**Red**	10
2	**Blue**	20
3	?	30

Extension

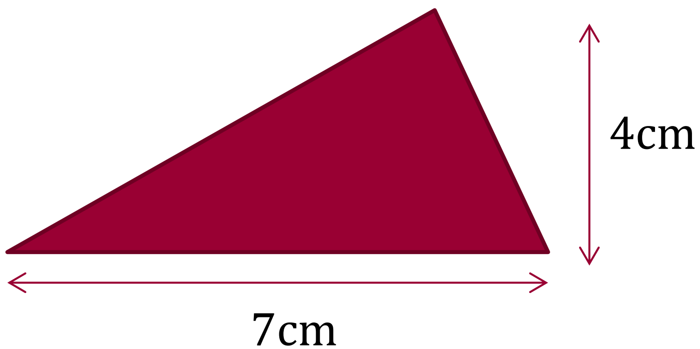
Possible extensions: If Mr. Byrne arrive with his 3 sons (triplets), and they each wanted a piece of gum, which had to be the same colour, what would be his maximum cost to ensure this happened?

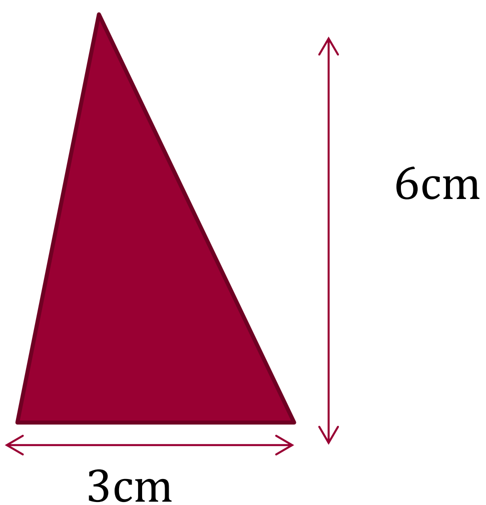
What if the bubble gum machine sold 3 different colours of gum, e.g. red, blue and green? How would this effect the out come of each case?

Adapting questions

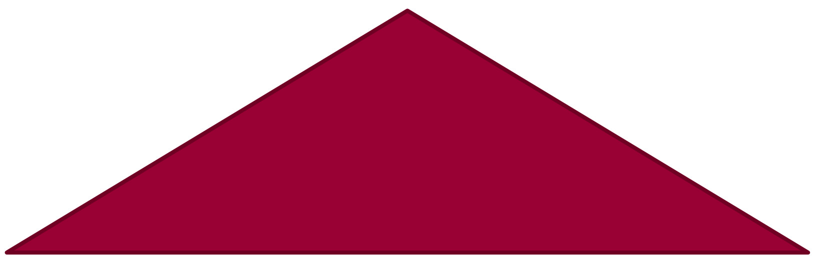
Solving a Problem

Find the area of these triangles





Problem Solving



Find the area of this triangle, taking any measurements you consider necessary.

How accurate is your answer?

How could you check your answer without repeating the same calculation?

Adapting questions

Solving a Problem

Draw graphs of the following equations;

$$y = 5$$

$$x + y = 4$$

$$y = 3x + 2$$

$$3x + 2y = 24$$

$$y = -3x + 2$$

$$y = \frac{x}{2}$$

Problem Solving

Write down a simple linear equation, e.g. $y = x + 5$ and draw its graph.

Give only the graph to your neighbour and ask them to reconstruct the original equation.

Now try to make up some harder examples

Pylon LCHL ⟶ JCOL

Two surveyors want to find the height of an electricity pylon.

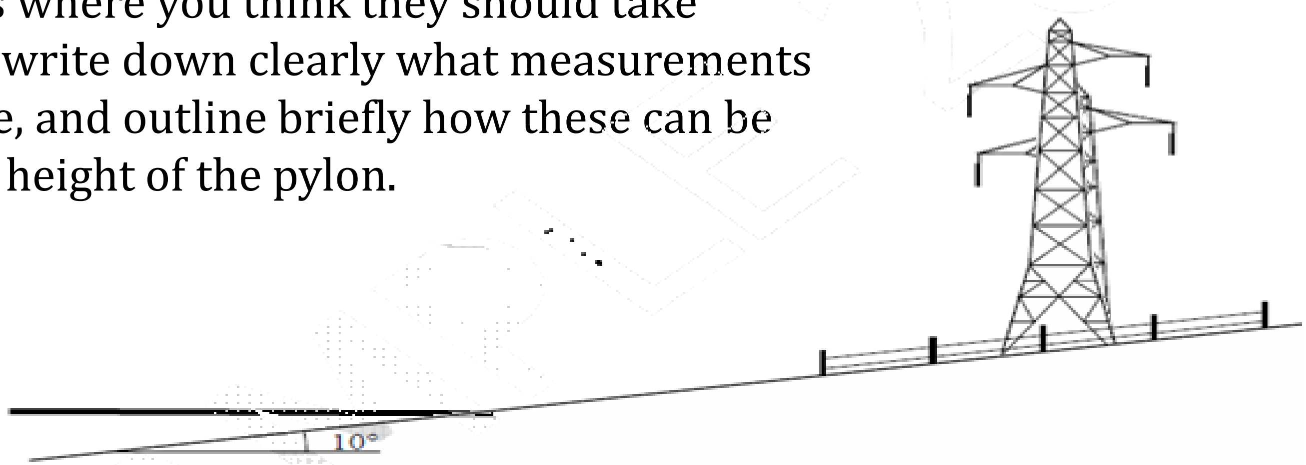
There is a fence around the pylon that they cannot cross for safety reasons. The ground is inclined at an angle.

They have a clinometer (for measuring angles of elevation) and a 100 metre tape measure.

They have already used the clinometer to determine that the ground is inclined at 10° to the horizontal.

(a) Explain how they could find the height of the pylon.

Your answer should be illustrated on the diagram below. Show the points where you think they should take measurements, write down clearly what measurements they should take, and outline briefly how these can be used to find the height of the pylon.



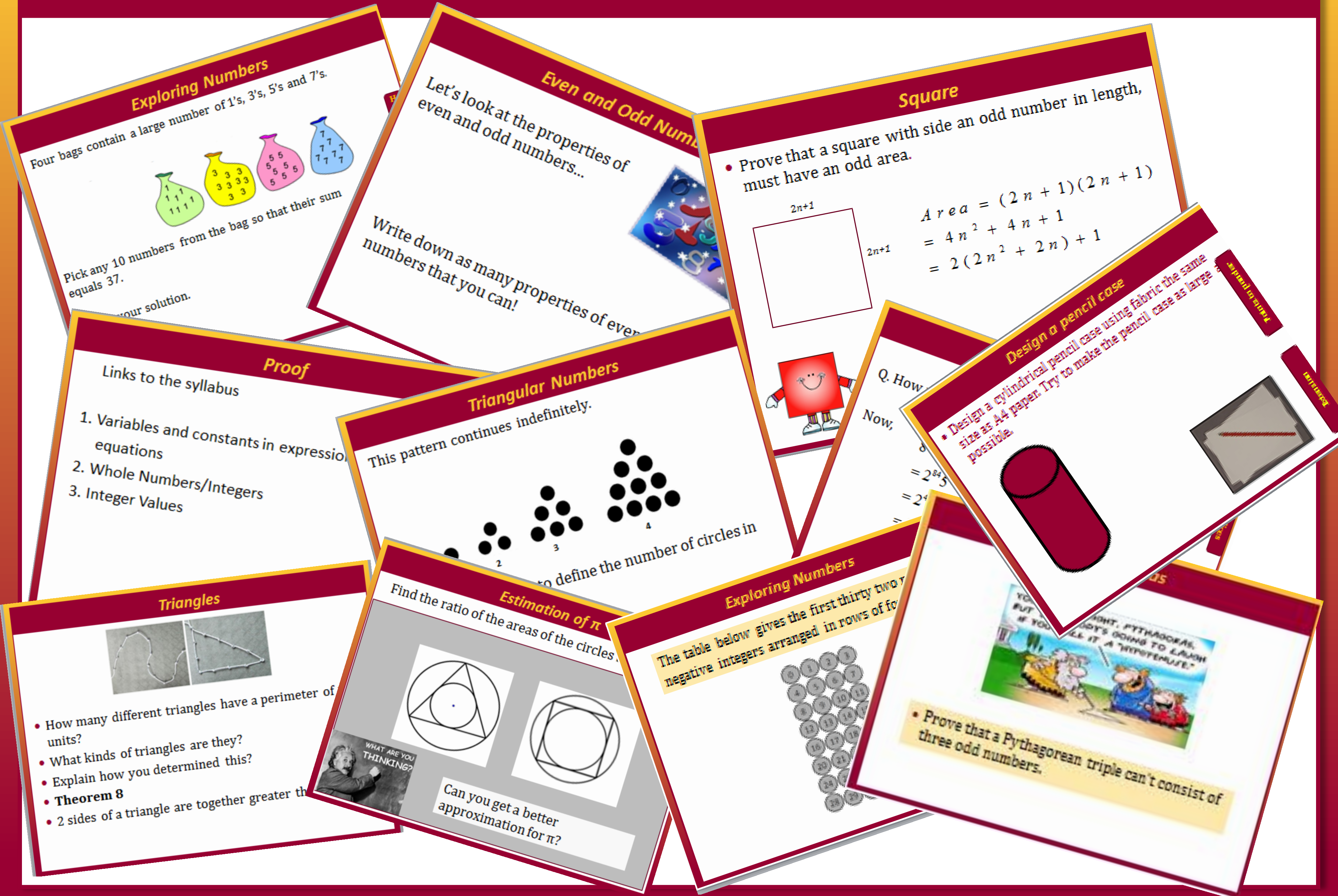
Pylon Junior Cert

Write down possible values for the measurements taken, and use them to show how to find
the height of the pylon. (That is, find the height of the pylon using your measurements, and
showing your work.)

Questions

- You must have a reason for asking questions.
- Students think about how they thought about it.
- The student voice is LEAST clearly heard in maths than in any other subject.

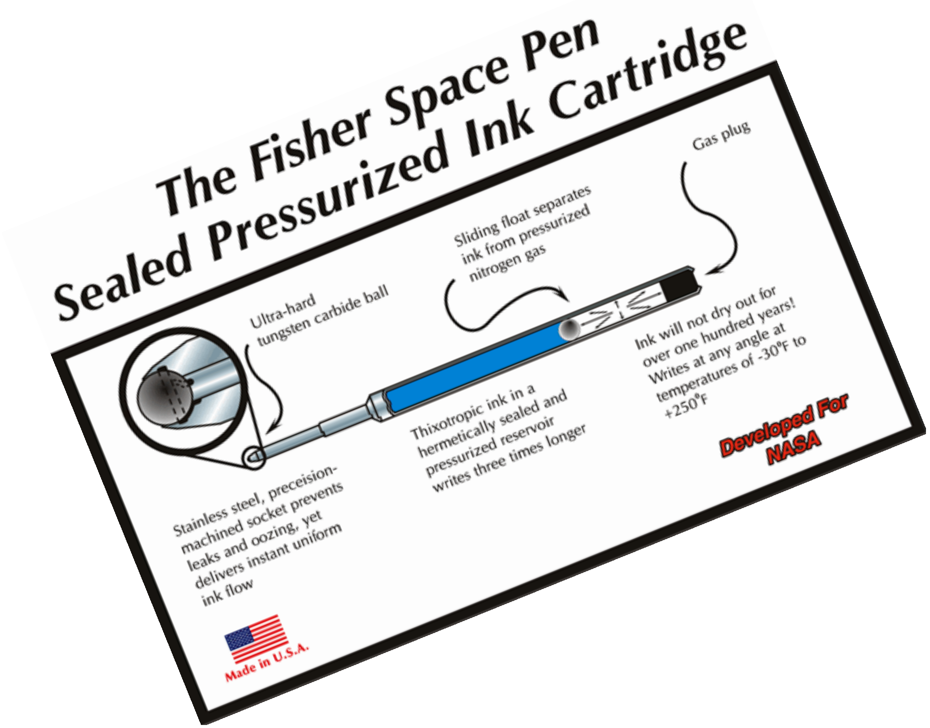
- Use at start of lesson to motivate it.
- Some don't have an obvious solution.
- The children you regard as best at maths aren't always the best.
- Rigor is necessary.


Exploring Numbers
Four bags contain a large number of 1's, 3's, 5's and 7's.
Pick any 10 numbers from the bag so that their sum equals 37.
Even and Odd Numbers
Let's look at the properties of even and odd numbers...
Write down as many properties of even
numbers that you can!
Square
Prove that a square with side an odd number in length, must have an odd area.
2n+1
2n+1
Area = (2n + 1)(2n + 1)
= 4n² + 4n + 1
= 2(2n² + 2n) + 1
Design a pencil case
Design a cylindrical pencil case using fabric the same size as A4 paper. Try to make the pencil case as large as possible.
Proof
Links to the syllabus
1. Variables and constants in expressions
equations
2. Whole Numbers/Integers
3. Integer Values
Triangular Numbers
This pattern continues indefinitely.
define the number of circles in
Triangles
How many different triangles have a perimeter of
units?
What kinds of triangles are they?
Explain how you determined this?
Theorem 8
2 sides of a triangle are together greater th
Estimation of π
Find the ratio of the areas of the circles
Can you get a better approximation for π?
WHAT ARE YOU THINKING?
Exploring Numbers
The table below gives the first thirty two
negative integers arranged in rows of fo
Prove that a Pythagorean triple can't consist of three odd numbers.

Mathematics

Problem solving

In the early years of the “Space Race” the USA spent millions of dollars creating a “Space” pen. One that their astronauts could use in a zero gravity environment, write on almost any surface and work in temperatures ranging from -100^{o}C to 100^{o}C



Problem solving

The Russians faced the same problem,

They used a PENCIL