Project Maths Workshop 9

Use & Value Connections

Name: ______________________

School: ______________________

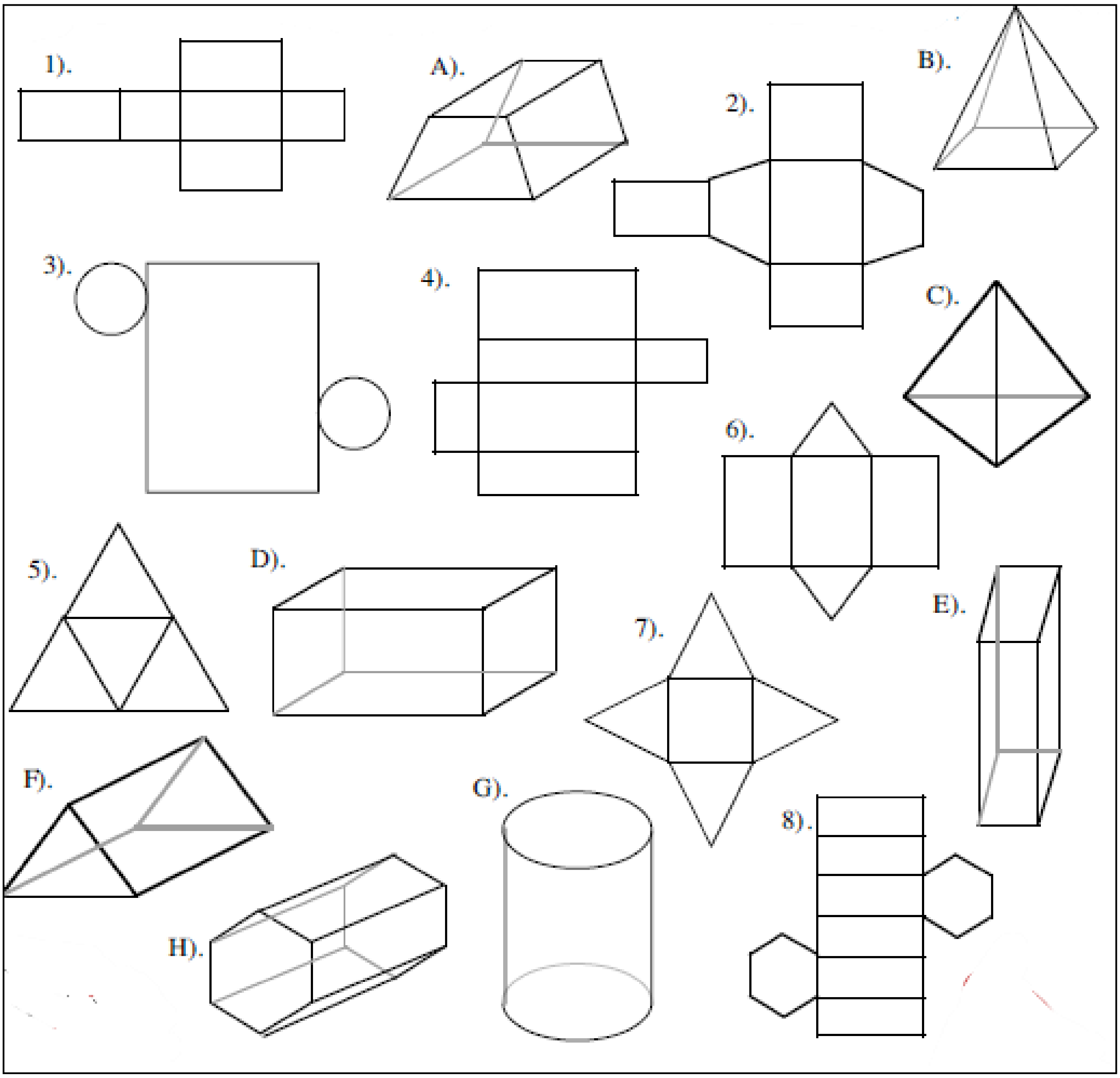
Project Maths Tionscadal Mata Development Team ©

Contents

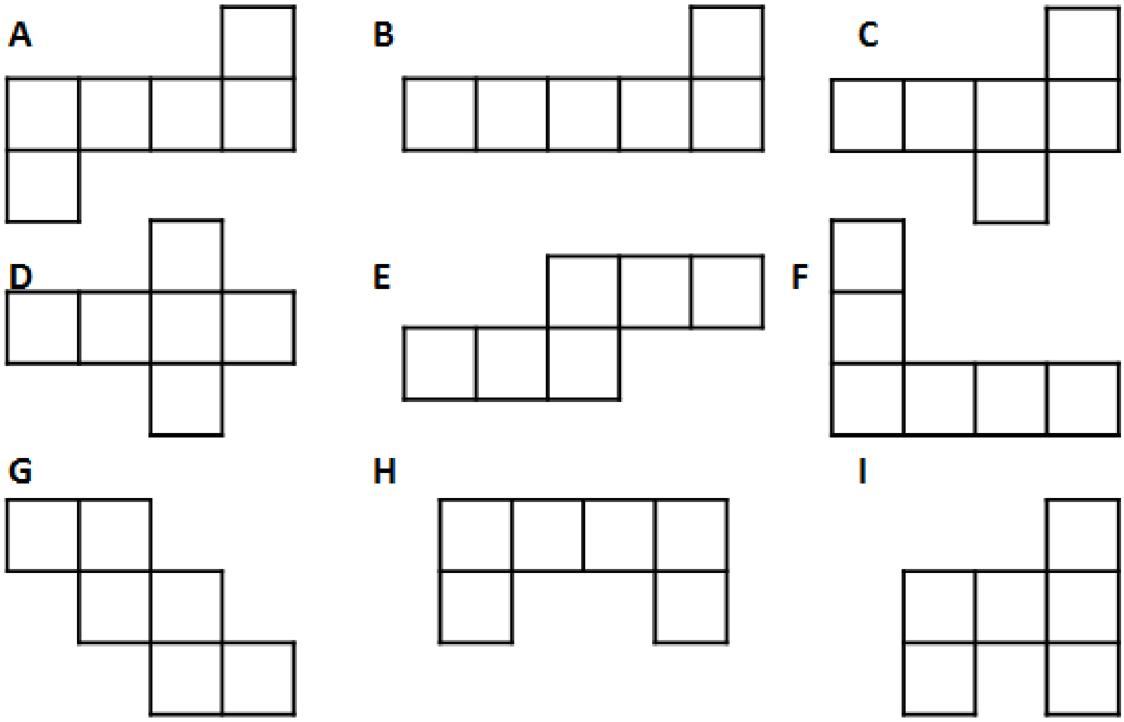
WS09.01 Nets

Q1. Match the net of the prism or pyramid with its 3D shape

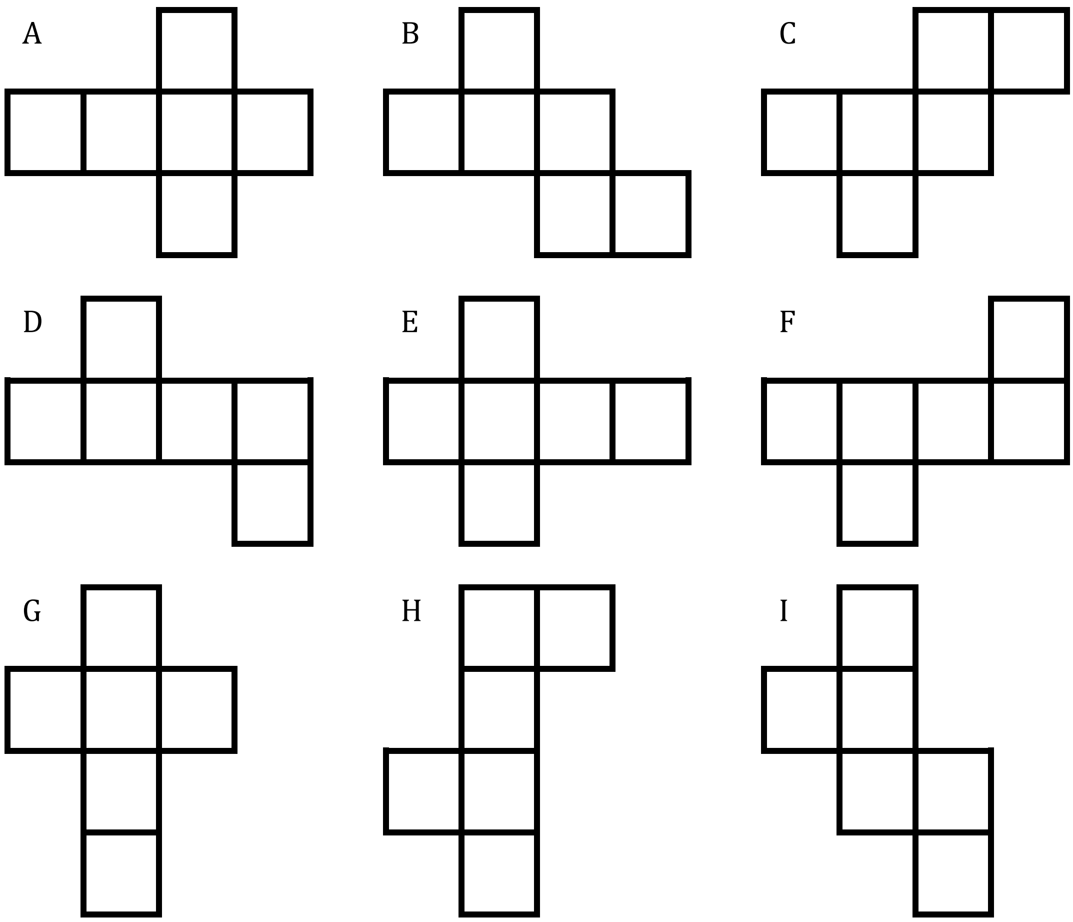
3D Shape	A	B	C	D	E	F	G	H
Net								



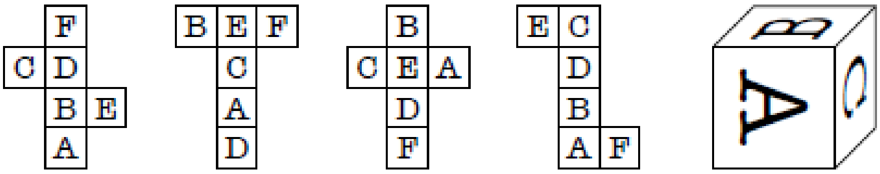
Q2. Which of the following are nets of a cube?

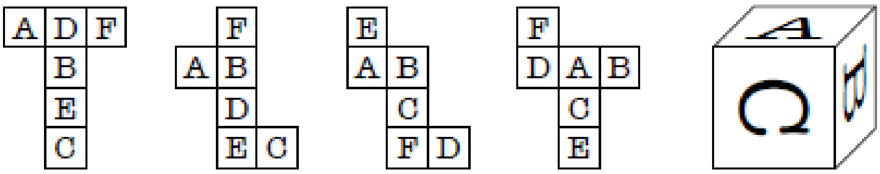


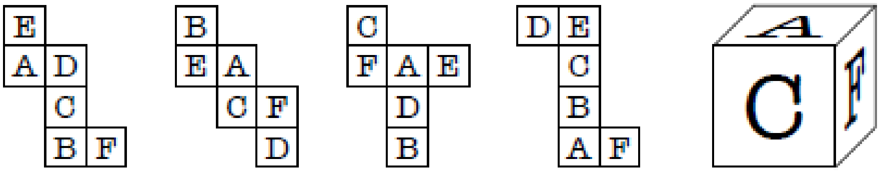
Q3. List the nets that are the same:

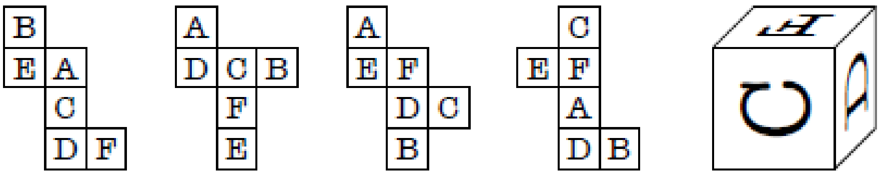


Q4. Find the mapping which can be folded to make the cube:









Activity 1

For each function, write in its correct derivative.

$5x$	
$5x+2$	
$5x-10$	
$x^2+\pi$	
x^2	
$\sin(x)$	
$\sin(x)-1.3$	
$\sin(x)+9$	
$\frac{1}{2}x^2$	
$\frac{1}{2}x^2-0.358$	

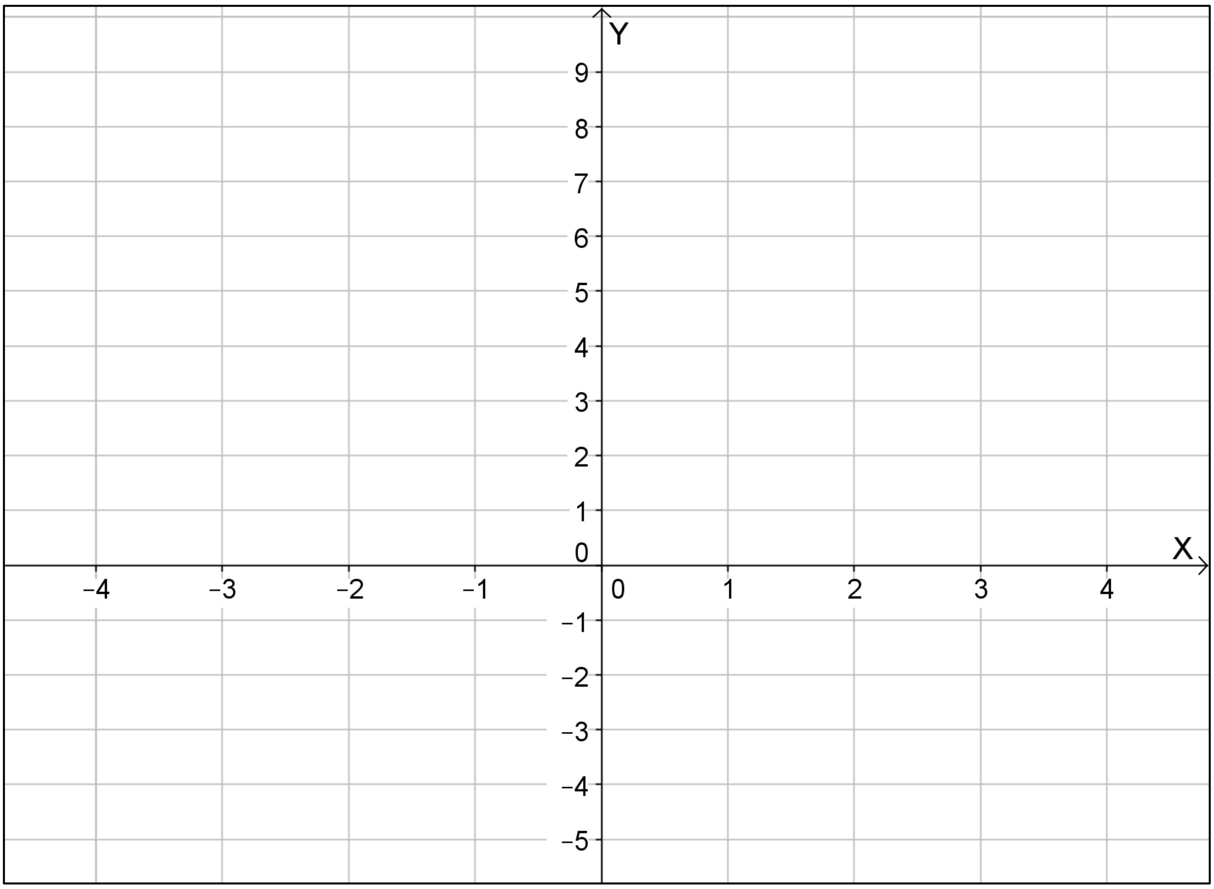
Activity 2

Find the anti-derivative of the function $f(x) = 3$ which passes through the point $(1, 5)$.

Q1. How is this question different to all the previous anti-derivative questions you have encountered?

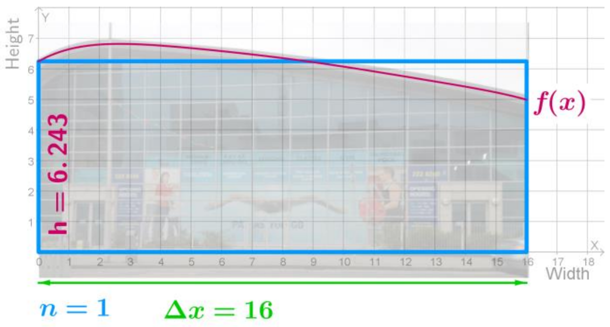
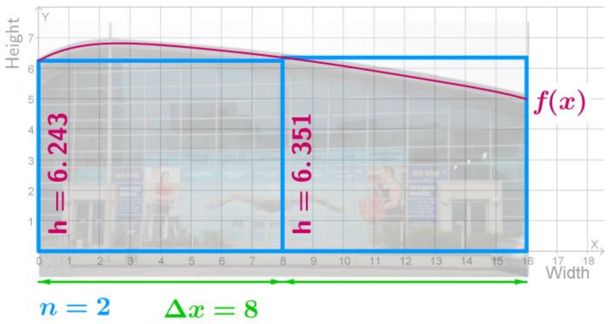
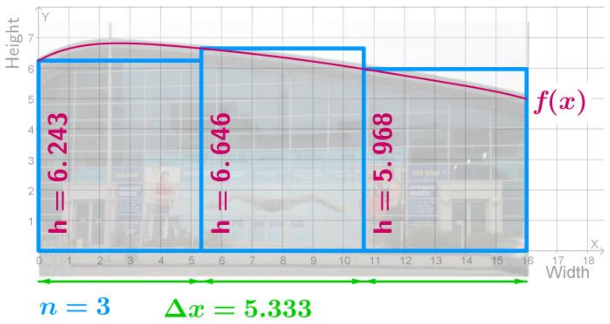
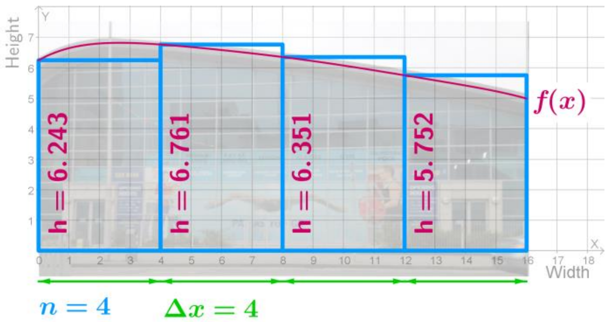
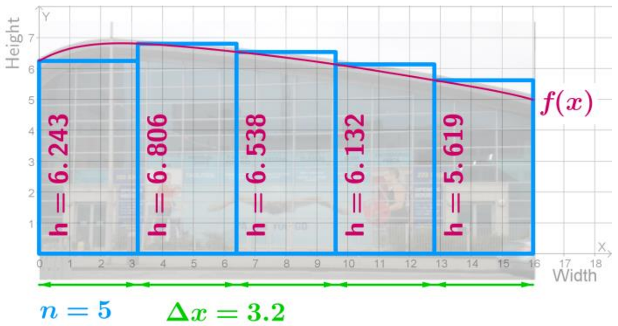
Q2. Find the indefinite form of the anti-derivative of $f(x) = 3$.

Q3. Represent the indefinite form of the anti-derivative graphically below by sketching the anti-derivatives for each of the following values of $C = \{-3, -2, -1, 0, 1, 2, 3\}$.



Q4. Identify the distinct anti-derivative you were asked to find.

Activity 3

		Area Calculation
(i)		
(ii)		
(iii)		
(iv)		
(v)		

Explain what happens to the width of the rectangles (Δx) as the number of rectangles (n) increases. Express this relationship using mathematical notation.

Description in words: As the number of rectangles increases, the width of the rectangles ______

As $n \rightarrow$ ______ , $\Delta x \rightarrow$ ______

Activity 4

Calculate $\int_2^5 (2x+1)dx$.

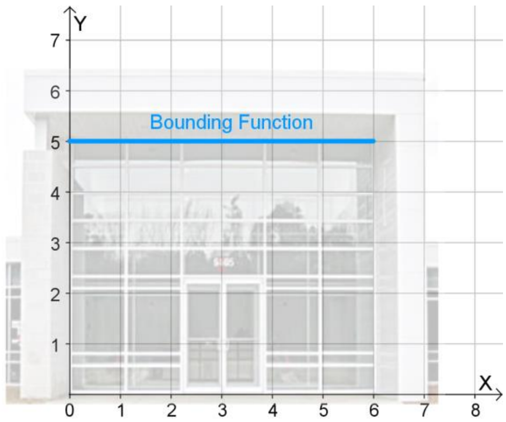
Q1.	In words describe what you are being asked to do.
Q2.	Using a suitable approach, complete the task.

Activity 5

Group A

Figure 1 shows the UCD Student Computer Centre.

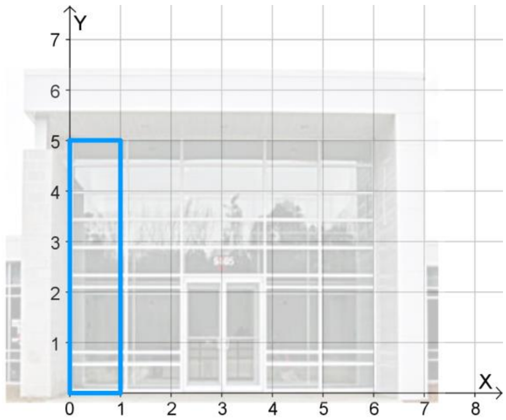
Figure 1 - The UCD Student Computer Centre.



Q1. Write down the function which describes the height of the building as we move from left ($x = 0$) to right ($x = 6$).

$h(x) =$

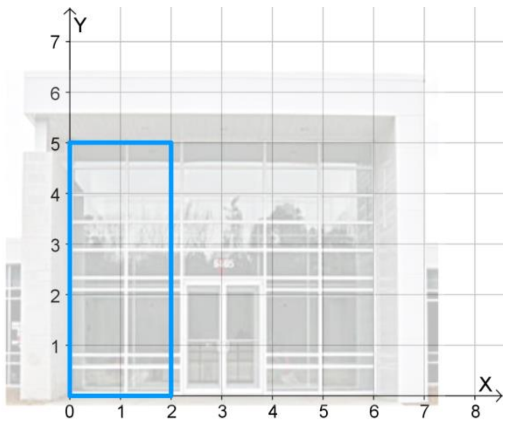
The area under the bounding function changes as we move from left to right. We will now investigate the relationship between the area of the building and its width.



Q2. By calculating the area of the rectangular piece of building shown, complete the statement:

When **the width of the rectangular piece is 1** unit, the area of the rectangle is:

$A =$



Q3. By calculating the area of the rectangular piece of building shown, complete the statement:

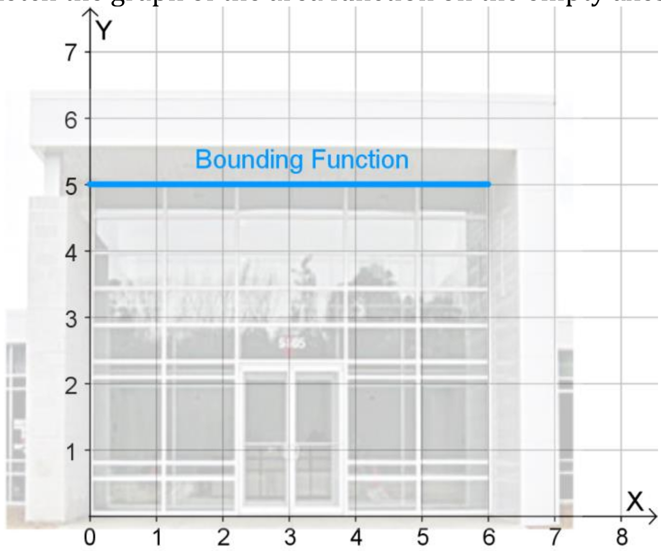
When **the width of the rectangular piece is 2** units, the area of the rectangle is:

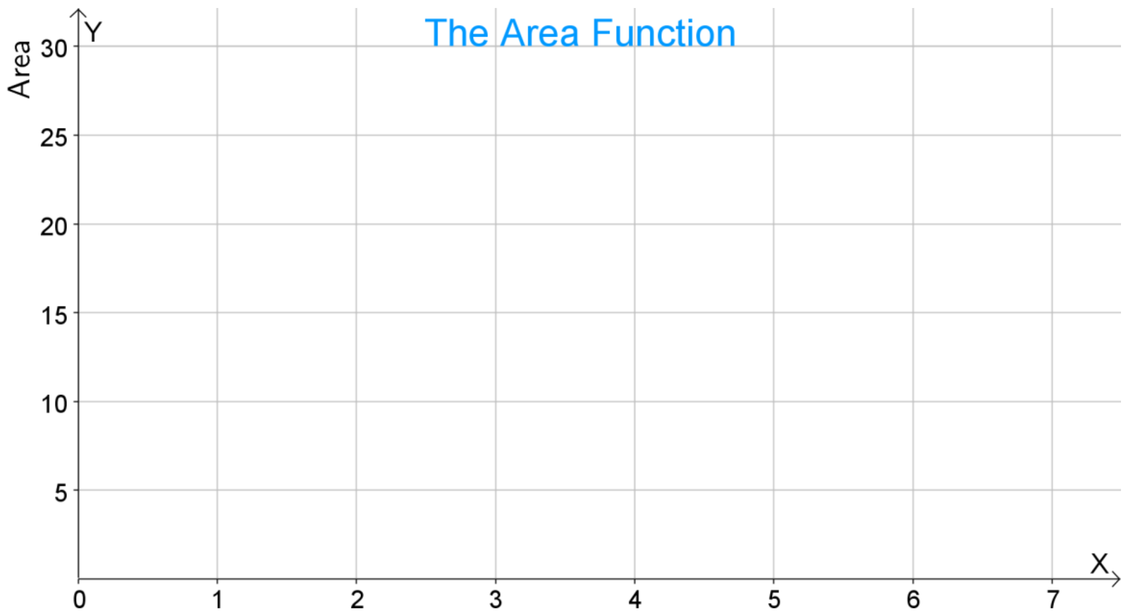
$A =$

Q4. Complete the table below using an approach similar to that used in Q2 and Q3.

x	Width	Height	Area	Pattern
0	0	5	0	$A = 5(0)$
1	1	5	5	$A = 5(1)$
2				$A =$
3				$A =$
4				$A =$
5				$A =$
6				$A =$
⋮	⋮	⋮	⋮	⋮
x			$A(x) =$	

Q5. Sketch the graph of the area function on the empty axes.



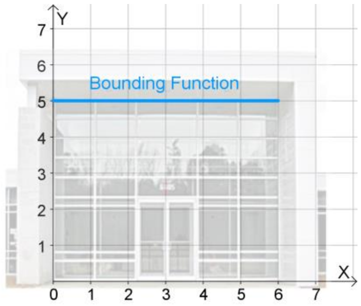
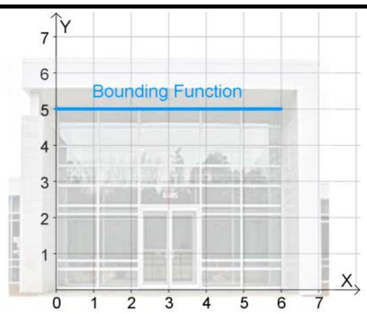
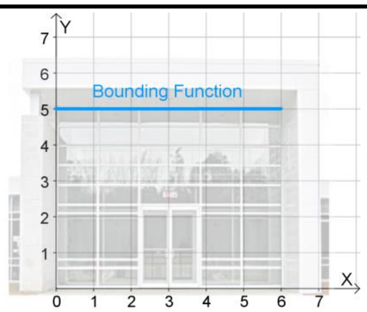


Q6. For each of the areas in the table below:

(a) Shade in the given area on the diagram.

(b) **<u>Use the area function</u>** to calculate the given area.

(c) Explain how the area function is used to calculate area.

<table>
<tr><th>Section of Building</th><th>Diagram</th><th>Area Calculation</th></tr>
<tr><td>From $x = 0$ up to $x = 2$.</td><td></td><td></td></tr>
<tr><td colspan="3">Explanation:</td></tr>
<tr><td>From $x = 0$ up to $x = 5$.</td><td></td><td></td></tr>
<tr><td colspan="3">Explanation:</td></tr>
<tr><td>From $x = 2$ up to $x = 5$.</td><td></td><td></td></tr>
<tr><td colspan="3">Explanation:</td></tr>
</table>

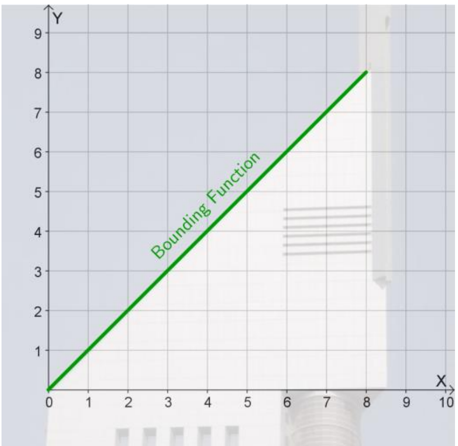
Q7. **(a)** In the space below write in the bounding function (from Q1 above) and the area function (from Q3 above).

Bounding Function	Area Function
$h(x) =$	$A(x) =$

(b) If you were presented only with the bounding function, is there a way in which you could determine the area function? Explain.

Group B
Figure 2 shows The Vu Bar in Dubai.

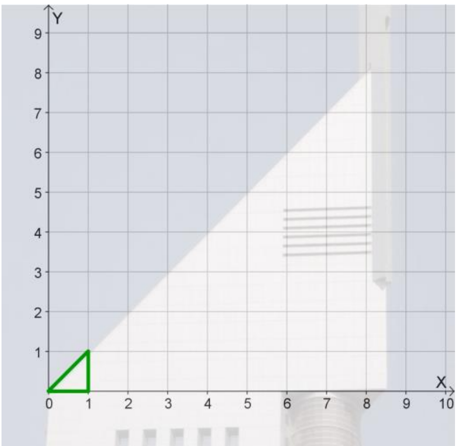
Figure 2 - The Vu Bar, Dubai.



Q1. The height of the building changes as we move from from left ($x = 0$) to right ($x = 8$). Write down the function which describes the changing height of the building.

$h(x) =$

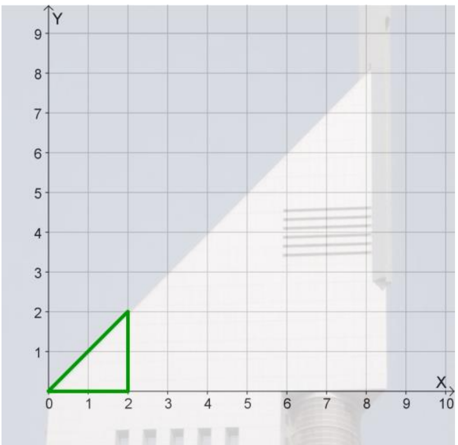
The area under the bounding function changes as we move from left to right. We will now investigate the relationship between the area of the building and its width.



Q2. By calculating the area of the triangular piece of building shown, complete the statement:

When **the width of the triangular piece is 1** unit, the area of the triangle is:

$A =$



Q3. By calculating the area of the triangular piece of building shown, complete the statement :

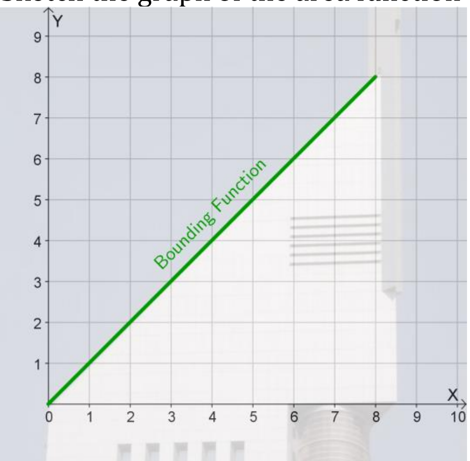
When **the width of the triangular piece is 2** units, the area of the rectangle is:

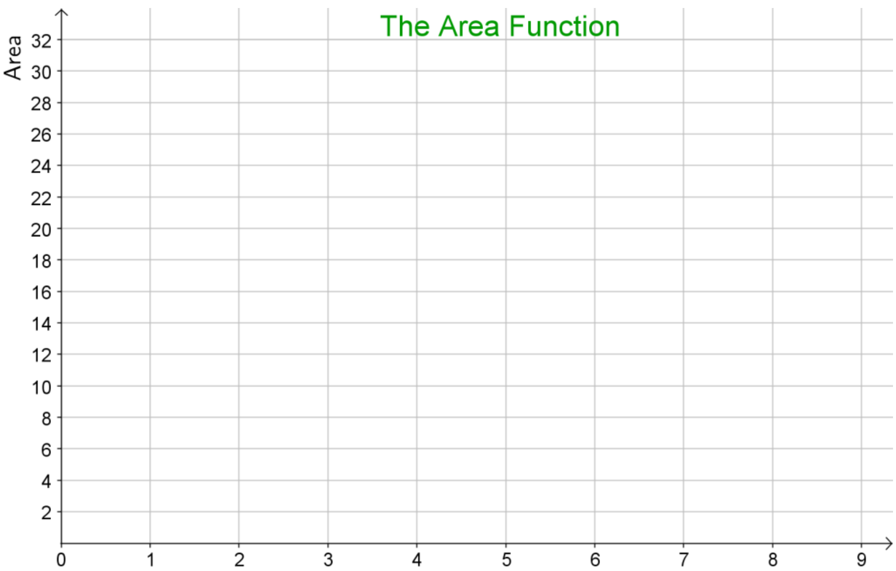
$A =$

Q4. Complete the table below using an approach similar to that used in Q2 and Q3.

x	Width	Height	Area	Pattern
0	0	0	0	$A = \frac{1}{2}(0)(0)$
1	1	1	0.5	$A = \frac{1}{2}(1)(1)$
2				$A =$
3				$A =$
4				$A =$
5				$A =$
6				$A =$
7				
8				
⋮	⋮	⋮	⋮	⋮
x			$A(x) =$	

Q5. Sketch the graph of the area function on the empty axes.



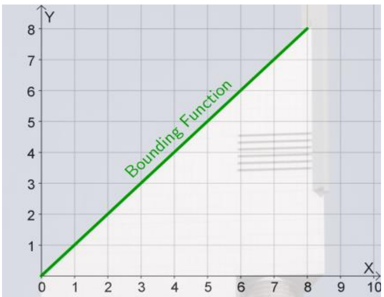
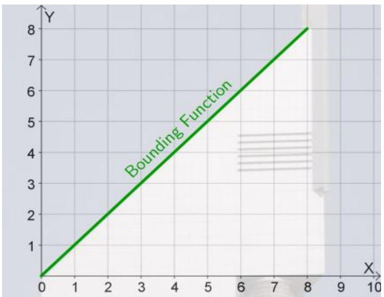
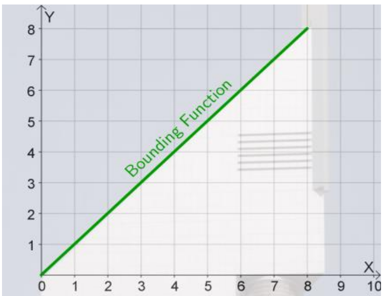


Q6. For each of the areas in the table below:

(a) Shade in the given area on the diagram.

(b) **<u>Use the area function</u>** to calculate the given area.

(c) Explain how the area function is used to calculate area.

Section of Building	**Diagram**	**Area Calculation**
From $x = 0$ up to $x = 3$.		
Explanation:		
From $x = 0$ up to $x = 5.5$.		
Explanation:		
From $x = 3$ up to $x = 5.5$.		
Explanation:		

Q7. **(a)** In the space below write in the bounding function (from Q1 above) and the area function (from Q3 above).

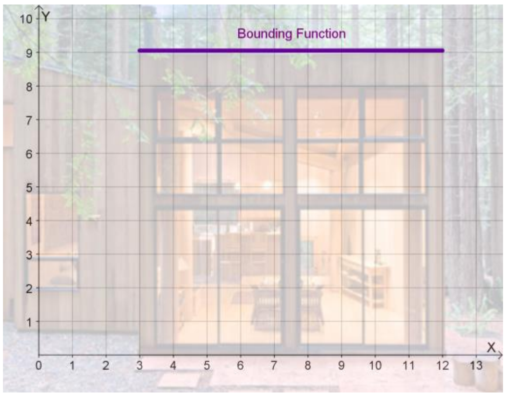
Bounding Function	Area Function
$h(x) =$	$A(x) =$

(b) If you were presented only with the bounding function, is there a way in which you could determine the area function? Explain.

Group C

Figure 3 shows a modern timber dwelling.

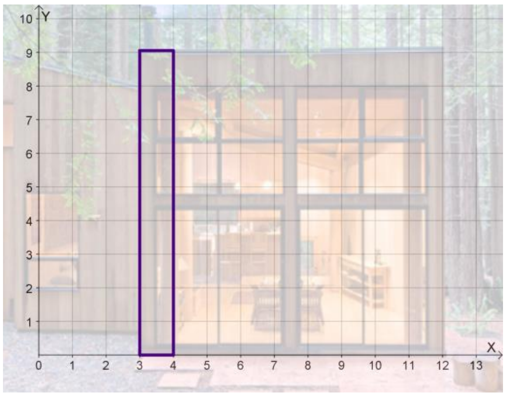
Figure 3 – Timber Dwelling.



Q1. Write down the function which describes the height of the building as we move from left ($x = 0$) to right ($x = 12$).

$h(x) =$

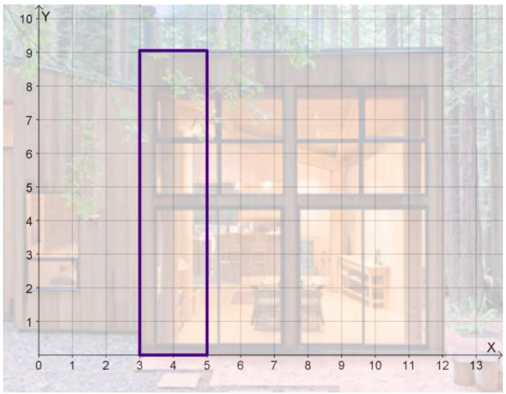
The area under the bounding function changes as we move from left to right. We will now investigate the relationship between the area of the building and its width.



Q2. By calculating the area of the rectangular piece of building shown, complete the statement:

When **the width of the rectangular piece is 1** unit, the area of the rectangle is:

$A =$



Q3. By calculating the area of the rectangular piece of building shown, complete the statement:

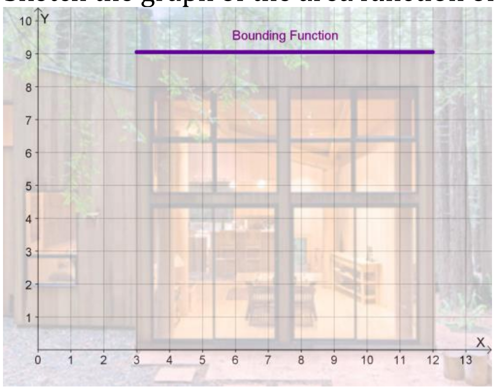
When **the width of the rectangular piece is 2** units, the area of the rectangle is:

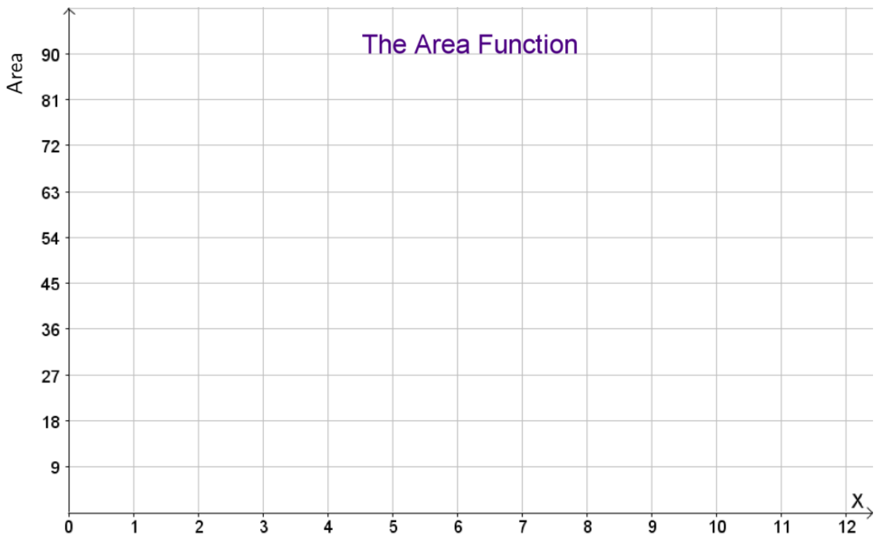
$A =$

Q4. Complete the table below using an approach similar to that used in Q2 and Q3.

x	Width	Height	Area	Pattern
3	0	9	0	$A = (9)(0)$
4	1	9	9	$A = (9)(1)$
5				$A =$
6				$A =$
7				$A =$
8				$A =$
9				$A =$
10				
11				
12				
⋮	⋮	⋮	⋮	⋮
x			$A(x) =$	

Q5. Sketch the graph of the area function on the empty axes.





Q6. For each of the areas in the table below:

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(b) **<u>Use the area function</u>** to calculate the given area.

(c) Explain how the area function is used to calculate area.

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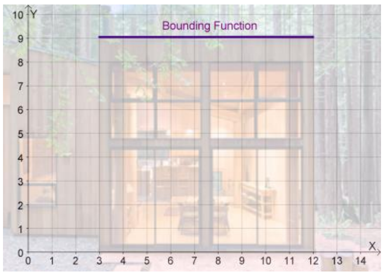
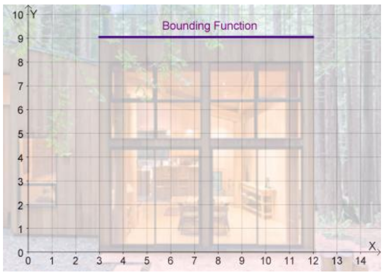
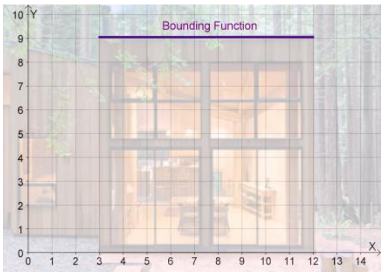
<tr><td>From $x = 3$ up to $x = 11$.</td><td>
</td><td></td></tr>
<tr><td colspan="3">Explanation:</td></tr>
<tr><td>From $x = 3$ up to $x = 6$.</td><td>
</td><td></td></tr>
<tr><td colspan="3">Explanation:</td></tr>
<tr><td>From $x = 6$ up to $x = 11$.</td><td>
</td><td></td></tr>
<tr><td colspan="3">Explanation:</td></tr>
</table>

Q7. **(a)** In the space below write in the bounding function (from Q1 above) and the area function (from Q3 above).

Bounding Function	Area Function
$h(x) =$	$A(x) =$

(b) If you were presented only with the bounding function, is there a way in which you could determine the area function? Explain.

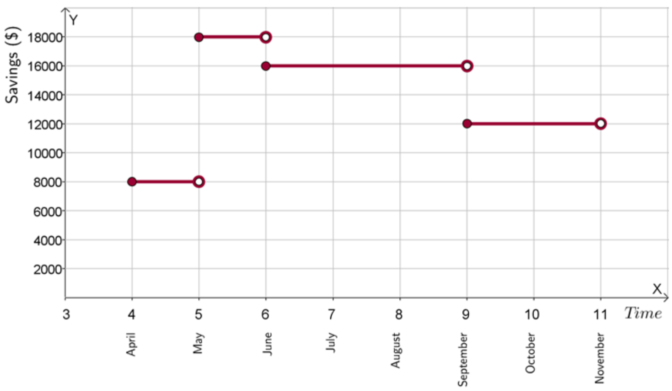
Activity 6

Question 1

Bernie has a savings account which she can use to lodge to or withdraw money. The table below shows the activity in the account over a 7 month period:

Time	Savings(€)
April	8000
May	18000
June	16000
July	16000
August	16000
September	12000
October	12000

Calculate the average amount of money in Bernie's account over the seven-month period.



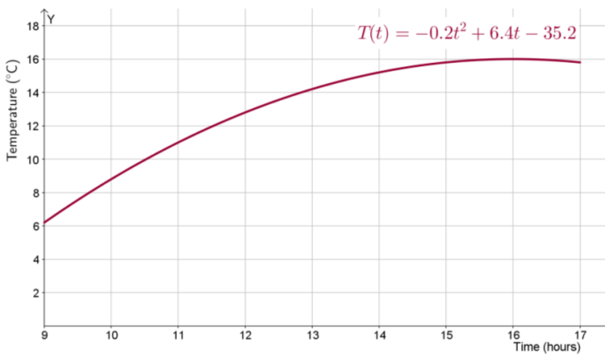
Question 2

On a certain day in Cork, air temperature was described by the following function:

$T(t) = -0.2t^2 + 6.4t - 35.2$ where $9 \leq t \leq 16$,

where T is temperature in °C and t is time since midnight in hours.

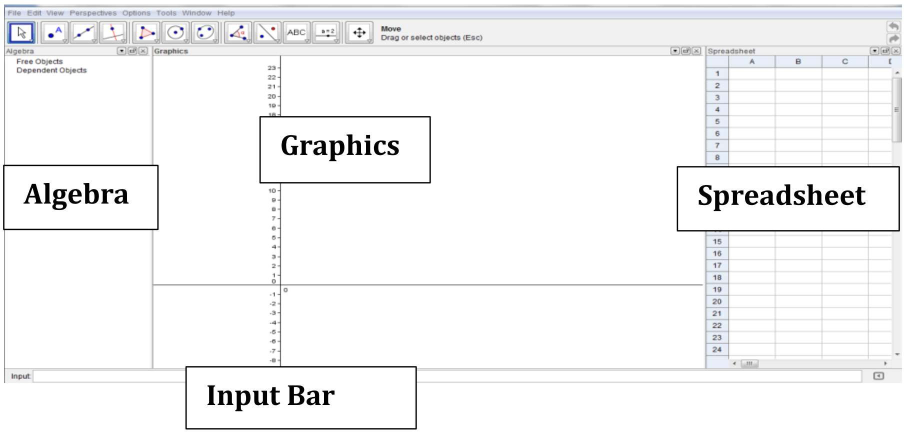
Calculate the average air temperature between 9 am and 4 pm



Activity 1: Introduction to GeoGebra

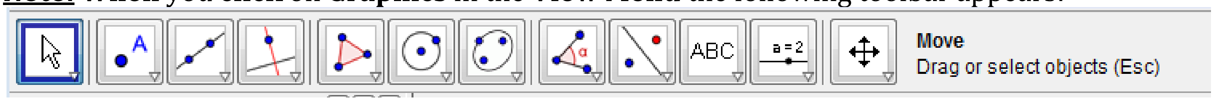
To download GeoGebra go to *www.geogebra.org* .

On opening GeoGebra the following window will appear.

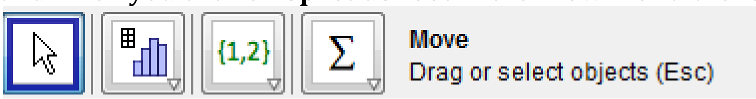


<u>Note:</u> If the **Spreadsheet** is not visible go to **View** and choose **Spreadsheet** and if the **Graphics** is not visible go to **View** and choose **Graphics.**

<u>Note:</u> When you **click** on **Graphics** in the **View Menu** the following toolbar appears:



and when you click on **Spreadsheet** in the **View Menu** the following toolbar appears:



In addition, when in the **Spreadsheet *view***, if one clicks on the right arrow  you get the **Toggle Style Bar**. This enables you to change the layout of the **Spreadsheet**.

<u>Note</u>: When drawing a function use $f(x) =$ rather than $y =$, because when $y =$ is used some of the commands from the Input Bar do not work.

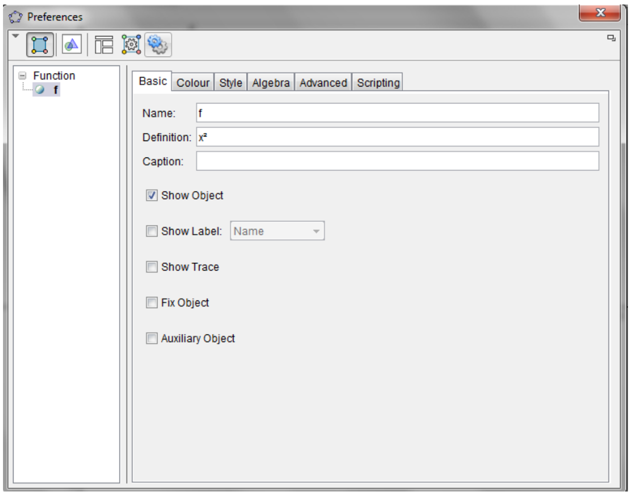
Activity 2: Graphing Multiple Functions

Input the following **Input Bar Commands** in the **Input Bar** and press **Return** on the keyboard.

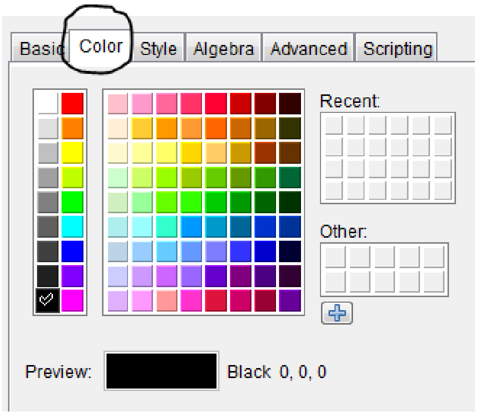
Function	Example	*GeoGebra* Input Bar Command
Linear	$f(x) = 4x - 3$	$f(x) = 4\ x - 3$
Quadratic	$g(x) = x^2 - x - 6$	$g(x) = x\text{^}2 - x - 6$
Cubic	$h(x) = x^3 - 4x^2 + 8x - 12$	$h\ (x\ ) = x\ \text{^}3 - 4\ x\text{^}2 + 8x - 12$
Exponential	$p(x) = 3^x$	$p(x) = 3\text{^}x$

Activity 3: To change the appearance of a graph of a function

1. Click on the graph of the function, right click and choose **Object Properties**. A new **Dialogue Box** appears.

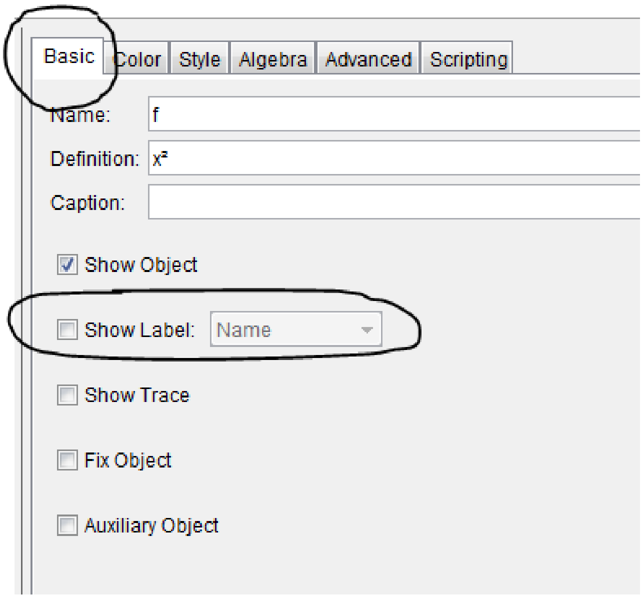


2. With the **Colour tab** open change the colour**.**



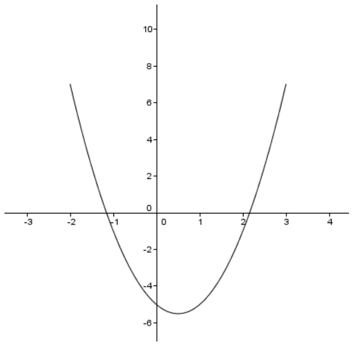
3. With the **Style tab** open use the **drop down menu** to change the style**.** and adjust the **Line Thickness** to the required width.

4. With the **Basic tab** open, click the **Show Label button** and choose **Name and Value** from the **drop down menu** to enable both the name of the graph of the function and its equation to be shown.



5. Click  at the top of the **Dialogue Box.**

Activity 4: To draw a function with a given domain and use the Intersect Two Objects tool



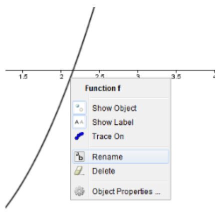
<u>For Example Question 6 (b) Junior Certificate Ordinary Level Paper 1 2013</u>

Draw the graph of the function $f: x \rightarrow 2x^2 - 2x - 5$ in the domain $-2 \leq \text{x} \leq 3$, where $\text{x} \in \mathbb{R}$.

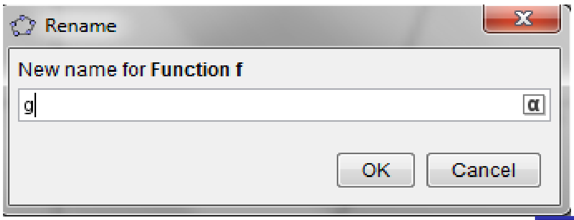
1. Go to **File** and choose **New Window**.
2. In the Input Bar type **Function[2x^2-2x-5,-2,3].**

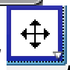
<u>Note</u>: If you are using the automatic **Function command** as in the diagram below, press the **Tab button** on your keyboard to move from Function to **Start x-Value** etc.

3. **Press Enter** on the keyboard.
4. As your function will be **automatically** called $\boldsymbol{f(x)}$**,** to rename it, right click on the graph of the function and choose **Rename.**



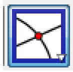
5. Replace ***f*** with ***g*** in the **Dialogue Box** that appears and press **OK**.



Note: ***To see the relevant area of this graph, select the Move Graphics tool*** ***, click the y axis and drag towards the origin.***

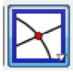
Question 6 (c) (i) Junior Certificate Ordinary Level Paper 1 2013

Use the graph drawn in 6(b) to estimate: The values of $2x^2 - 2x - 5$ when $x = 0.5$.

1. In the Input Bar type $\boldsymbol{x = 0.5}$ and press **Enter**.
2. Select the **Intersect Two Objects** tool  (in the second **drop-down menu** from the **left** on the **Graphics toolbar**) and click on the graph of the function g and the line $\boldsymbol{x = 0.5}$.
3. The co-ordinates of the point of intersection appear in the **Algebra View**.

Question 6 (c) (ii) Junior Certificate Ordinary Level Paper 1 2013

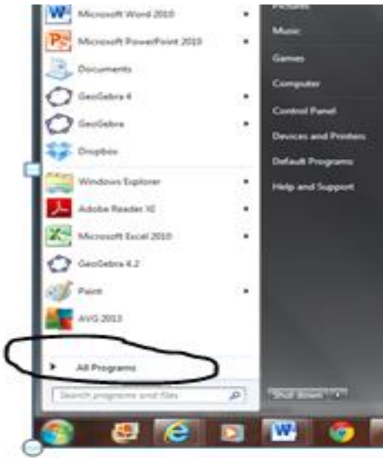
Use the graph drawn in 6(b) to estimate: The values of x for which $g(x) = 0$.

1. Select the **Intersect Two Objects** tool and click on the graph of the function g and the x axis.
2. The co-ordinates of the points of intersection appear in the **Algebra View**.

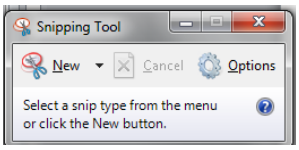
Alternatively: Type **Root[g]** in the **Input Bar** and press return

Activity 5: To Transfer a diagram made in GeoGebra to Word or PowerPoint

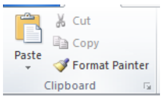
1. Draw a function (or whatever diagram is required) in **GeoGebra**.
2. Click on the **Start button**  at the bottom left hand side of your computer's screen**.**



3. Go to **All Programs, Accessories** and **Snipping Tool**.
4. A new **Dialogue Box** appears.



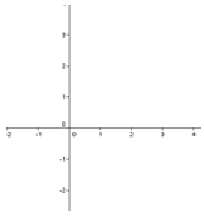
5. Click **New** and outline the area you want in your picture.

6. Open **Word** or **PowerPoint** and click **Paste** 

or click **Control** and **v** simultaneously on your keyboard.
7. To resize this picture, click on the picture and drag the dots on the corners of the pictures in or out as required.

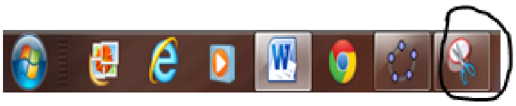
8. This picture can be centred by clicking  or press **Control** and **e** simultaneously on your keyboard.

9. To wrap the picture in text, right click the picture, choose **Wrap Text** and follow the arrow to the right to choose the **different layouts**.

This picture is wrapped through  the text.

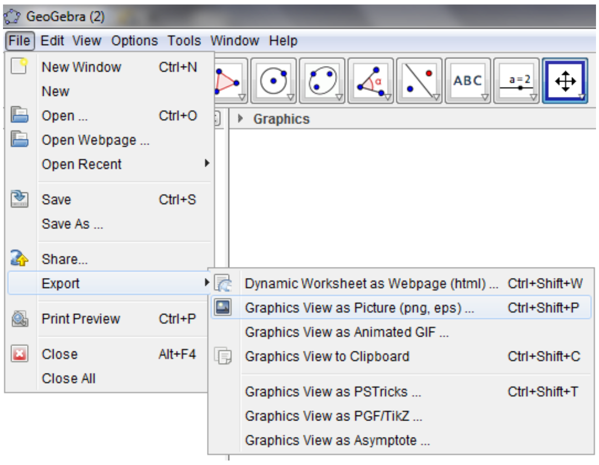
10. Other changes can be made to this picture by right clicking the picture and choosing **Object Properties**.

<u>Note</u>: To **pin** the **Snipping Tool** to the **Task Bar**, go to **All Programs, Accessories** and **Snipping Tool**, **right click** on **Snipping Tool** and choose **Pin to Taskbar**.

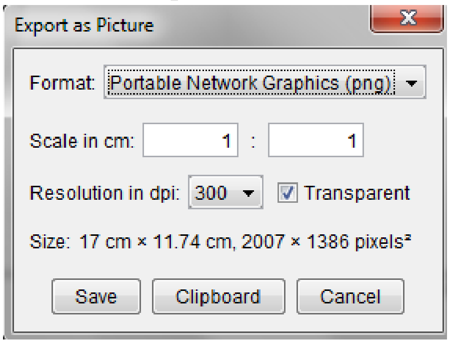


Activity 6: An Alternative to using the Snipping Tool in GeoGebra (Gives better picture quality.)

1. **Go** to **File**, **Export** and choose **Graphics View** as **Picture**.



2. Complete the new **Dialogue Box** and click **Clipboard.**

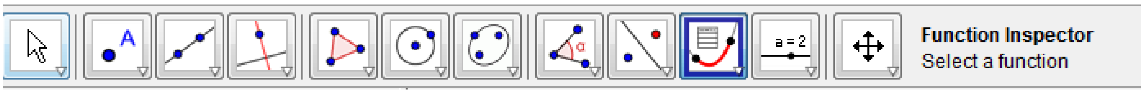


3. Open **Word or PowerPoint,** paste and adjust like any other picture.

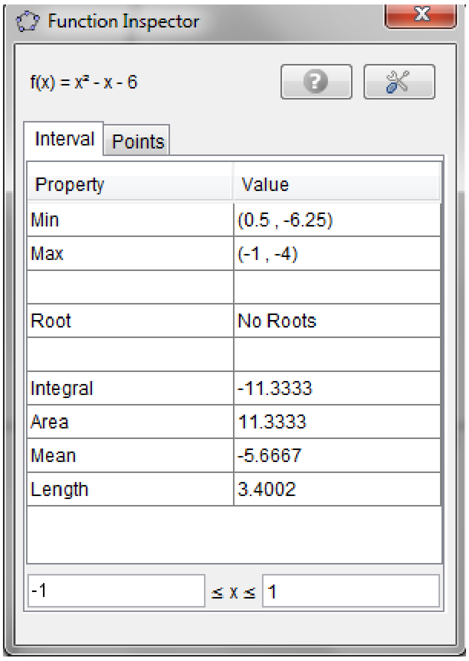
Activity 7: Function Inspector Tool

1. Draw your function in the usual way. For example in the Input Bar type **f(x) = x^2-x-6**.

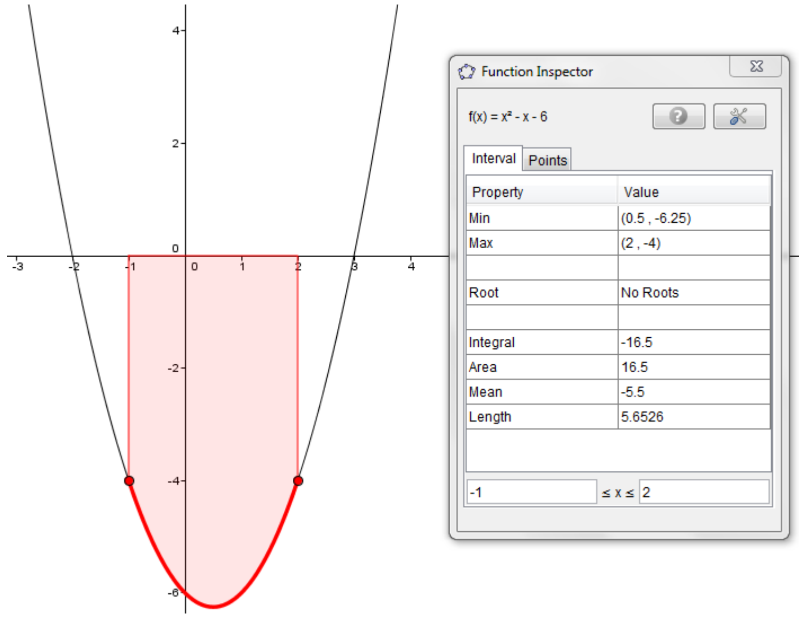
2. Click on the **Function Inspector Tool**  in the **third Drop-Down Menu** from the right on the **Graphics Toolbar**.



3. Click on the graph of the function to activate **Function Inspector** and a new dialogue box appears.



4. With the **Interval tab open** select the interval you want **to examine** for example, by typing –1 to 2in the active window at the bottom of the tab. After each change for example you change the '1' in the right-hand window to 2 *you must press the Enter button on your keyboard after each change for it to take effect.*



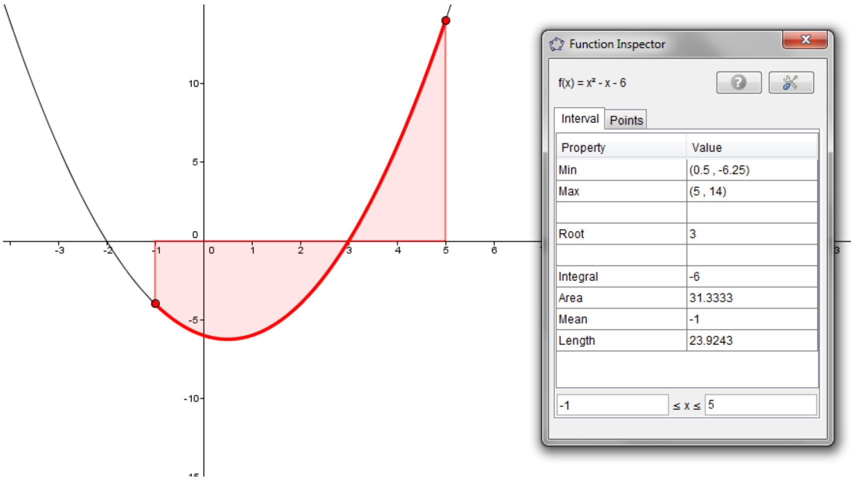
5. Click and drag the red dot(s) and watch how the area, the integral and the other values in the **Interval tab**. change.

Note: The Mean is the Average Value of the function.
Note: The Min. and Max. values given are the minimum and maximum values in the range being investigated.

<u>Explorations</u>:

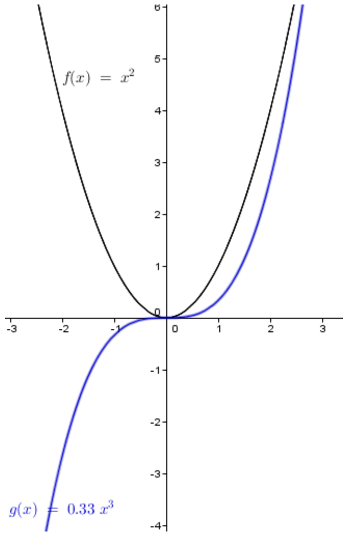
Why does it say this function has no roots?

Set the **interval** from -2 to 5. Why is the area now different from the integral?



Activity 8: To draw graph of the Integral of a function

1. Draw the graph of the function for example **f(x)=x^2**.
2. In the **Input Bar** type **Integral[f].**

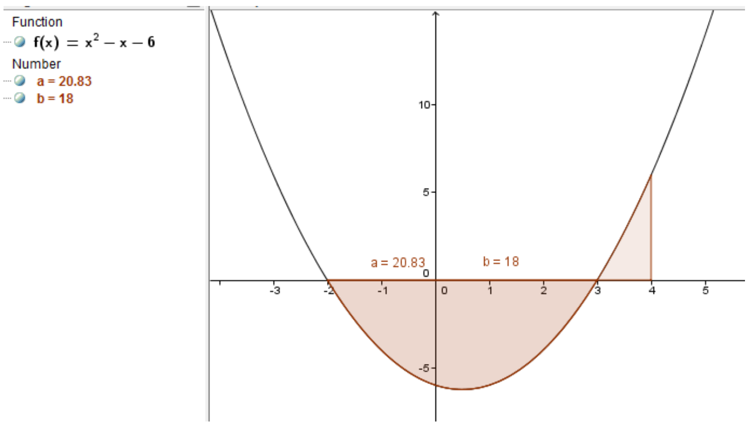


<u>Note</u>: This method takes the *Constant of Integration* to be *zero*.

Activity 9: Using the Input Bar to find the Integral of a function in an interval

1. Draw the graph of the function for example $\mathbf{f(x) = x^2 - x - 6}$
2. In the **Input Bar** type **Integral[f,-2,3]** and press **Enter**.
   (*Notice the negative answer.)*

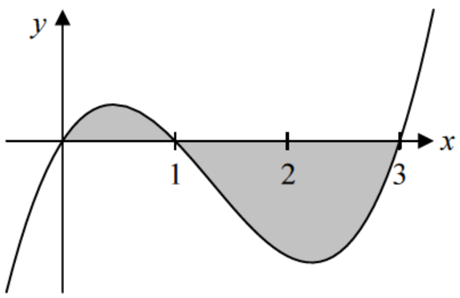
Now in the Input Bar type **Integral[f,-2,4],** what do you notice about the value of the Integral? Explain why this happened.



LCHL 2011: Q7 (b).

The curve $y = 12x^3 - 48x^2 + 36x$ crosses the x-axis at $x = 0$, $x = 1$ and $x = 3$, as shown.

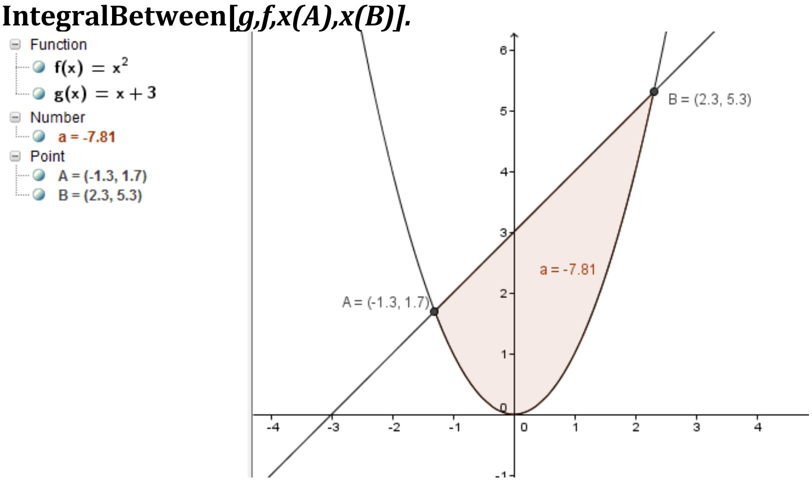
Calculate the total area of the shaded regions enclosed by the curve and the x-axis.



Activity 10: To find the area between two curves

1. Draw the graph of the function for example $(x) = x^2$.
2. Draw the graph of the function $g(x) = x + 3$.
3. Use the **Intersect Two Objects** tool to find the points of intersection between the two functions. The co-ordinates of points A and B appear in the Algebra view.
4. In the Input Bar type ***Integral[f,x(A),x(B)]*****.** This is represented by the value *a* in the **Algebra View.**
5. In the Input bar type ***Integral[g,x(A),x(B)]*****.** This is represented by the value *b* in the **Algebra View.**
6. In the Input Bar type $\boldsymbol{c = b - a}$**.** *c* is the area between the curves.

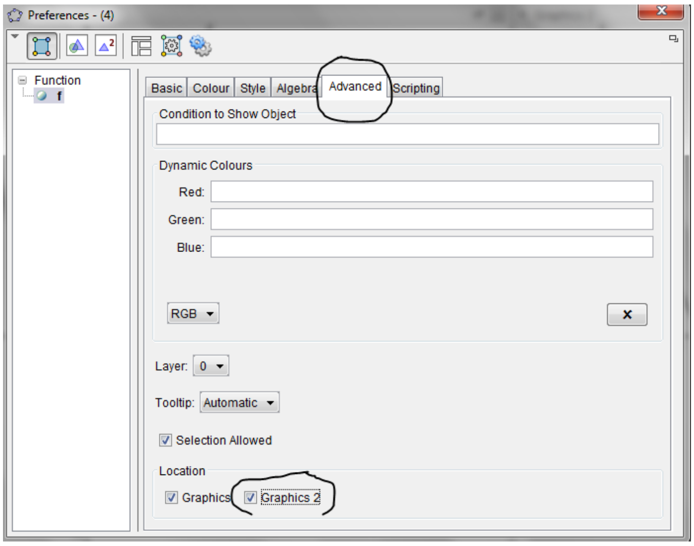
Note: Steps 4-6 can be replaced by:
In the Input Bar type ***IntegralBetween[g,f,x(A),x(B)].***

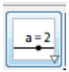


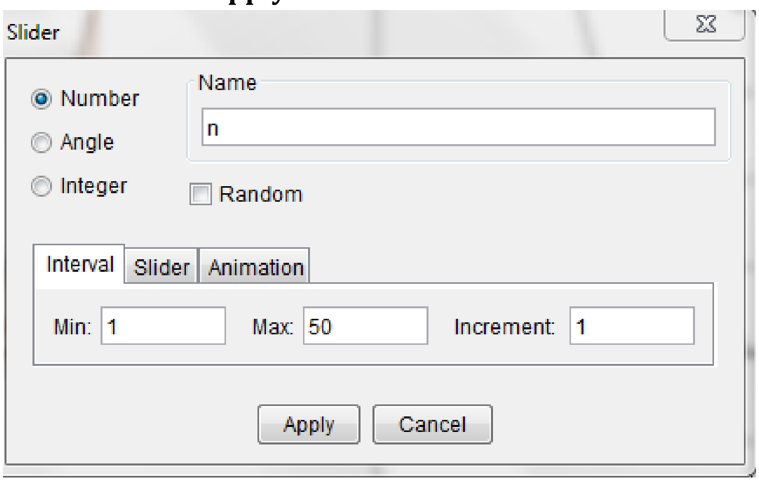
Activity 11: Using the two Graphics Views

Use the two Graphics views to find the Area under a curve by *(i)* the Integral method and *(ii)* the Trapezoidal Rule.

1. Go to **File** and choose **New Window**.
2. Draw the graph of your function in the usual way. For example in the **Input Bar** type **f(x) = x^2**.
3. Go to **View** and select **Graphics 2**. If the two Graphics views are not aligned right click on the **Graphics View** and choose **Standard View**.
4. Select the graph of your function, right click and choose **Object Properties.**
5. With the **Advanced tab** open, click **Graphics 2**.



6. Click  at the top of the **Dialogue box.**
7. **Click** on the **Graphics 1 View** and find the integral of the function between 0 and 2 as in the Activity 9 above.
8. Click on the **Graphics 2 View.**
9. Select the **Slider tool** . Click on the **Graphics 2 View** and create a **slider** called n with **Min:** =1, **Max:** =50 and **Increment:** = 1. Click **Apply**.



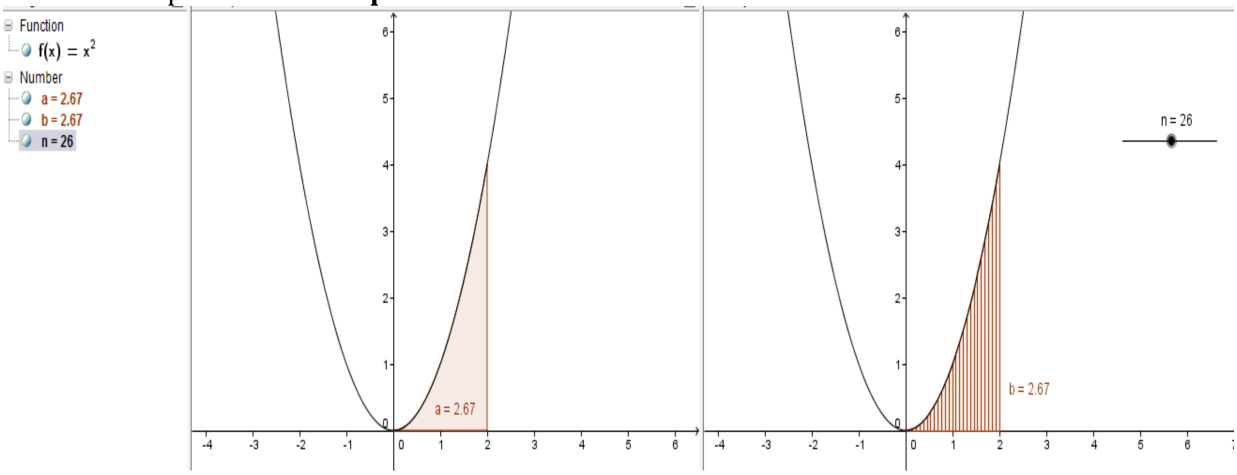
10. In the **Input Bar** type **b= TrapeziumSum[f,0,2,n].**

<u>**Note**</u>: **TrapeziumSum** is replaced by **TrapezoidalSum**, if the **GeoGebra Language** is set to **English(US)** instead of **English(UK).** To change the **GeoGebra Language** go to **Options**, **Language** and follow the arrows.

11. Move the **slider n** and as **n** gets larger check the relationship between the integral and trapezium area.

Note: The value for the **Trapezium sum** should eventually have the same value as the integral value when n increases.

Note: To get more accurate area values go to **Options**, **Rounding** and **choose** for example **10 Decimal places**.



Can you suggest other uses for the **two Graphics Views**?

Activity 12: To fit a graph to a list of points that are shown on the Spreadsheet view

1. Go to view and choose **Spreadsheet**.
2. Insert the *x* co-ordinates of the points in **Column A** and the *y* co-ordinates in the **column B**.

Spreadsheet

	A	B
1	-3	10
2	-2	5
3	-1	2
4	0	1
5	1	2
6	2	5
7	3	10

3. Highlight the two columns of data in the **Spreadsheet**, right click, choose **Create** and **List of points**.
4. In the **Input Bar** type **Fitpoly[list1,2],** if the list is list1 and you require a curve of degree 2 for example.

Note: If you require an exponential curve, input the co-ordinates of the points in the **Spreadsheet view** and create a list as above and then type **FitExp[list1]** in the **Input Bar,** if the list is list1.

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(b) A sprinter's velocity over the course of a particular 100 metre race is approximated by the following model, where v is the velocity in metres per second, and t is the time in seconds from the starting signal:

$$v(t)=\begin{cases}0, & \text{for } 0\le t<0{\cdot}2\\ -0{\cdot}5t^2+5t-0{\cdot}98, & \text{for } 0{\cdot}2\le t<5\\ 11{\cdot}52, & \text{for } t\ge 5\end{cases}$$

Note that the function in part **(a)** is relevant to $v(t)$ above.

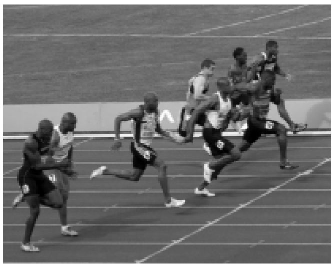
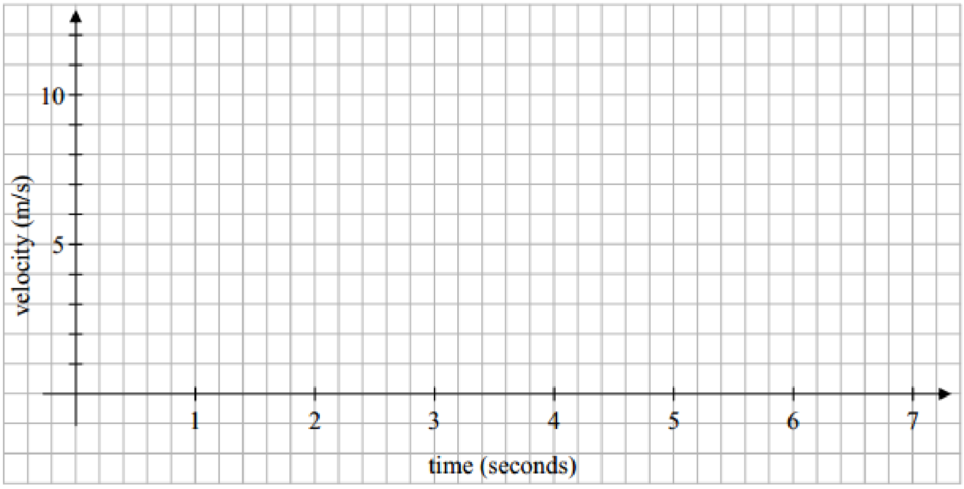


Photo: *William Warby*. Wikimedia Commons. CC BY 2.0

(i) Sketch the graph of v as a function of t for the first 7 seconds of the race.



(ii) Find the distance travelled by the sprinter in the first 5 seconds of the race.

(iii) Find the sprinter's finishing time for the race. Give your answer correct to two decimal places.

1. In the **Input bar** type **Function[0,0,0.2].**
2. In the **Input bar** type **Function[-0.5x^2+5x-0.98,0.2,5**].
3. Create a **slider** called lastpartofrace with **Min:** 5, **Max:** 15 and **Increment:** 0.01.
4. Adjust the **slider** so that it has a value of 15.
5. In the Input bar type **Function[11.52,5,lastpartofrace].**
6. In the Input bar type **Integral[f,0,0.2**].
7. In the Input bar type **Integral[g,0.2,5].**
8. In the Input bar type **Integral[h,5,lastpartofrace]**.
9. In the Input bar type **TotalDistance=a+b+c**
10. Adjust the **slider lastpartofrace** until **TotalDistance** is approximately 100.